210. *On the Generalized Korteweg-de Vries Equation*

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1. **Introduction.** The nonlinear dispersive equation of the type

$$u_t-(f(u))_x+\delta u_{xxx}=0, \qquad \delta>0 \tag{1.1}$$

is the generalization of the Korteweg-de Vries (KdV) equation

$$u_t-uu_x+\delta u_{xxx}=0, \qquad \delta\neq 0 \tag{1.2}$$

and is closely related to the study of anharmonic lattices, see [1].

In [2], A. Sjöberg proved, by the method of the semi-discrete approximation, existence and uniqueness of the global classical solutions of the KdV equation for an appropriate initial condition and periodic boundary condition. In [3], T. Mukasa and R. Iino extended Sjöberg's results to the simplest generalized KdV equation

$$u_t-u^2u_x+\delta u_{xxx}=0, \qquad \delta\neq 0. \tag{1.3}$$

By the method of parabolic regularization, R. Teman [4] obtained the existence and uniqueness theorems of the global weak solutions of the KdV equation for an appropriate initial condition and periodic boundary condition. In [5], Y. Kametaka proved existence and uniqueness of the global classical solutions of the Cauchy problem for the KdV equation and the simplest generalized KdV equation.
In [6], K. Masuda studied the Cauchy problem for the equation of the type

$$u_t-(u^p/p)_x+\delta u_{xxx}=0,\ p=1,2,3,4, \qquad \delta\neq 0. \tag{1.4}$$

In this note our aim is to extend those results to the generalized KdV equation (1.1) under the appropriate conditions imposed on $f(u)$, which include the cases $f(u)=u^p$, where p is an arbitrary odd number, and $f(u)=e^u$.

The plan of this paper is the following: In Section 2, we study the Cauchy problem for the nonlinear dissipative-dispersive equation

$$u_t-(f(u))_x+\delta u_{xxx}=\mu u_{xx}, \qquad \delta,\mu>0, \tag{1.5}$$

and establish the global existence theorems of the Cauchy problem for the equation (1.1) by letting μ tend to 0. Section 3 is devoted to present a semi-discrete approximation of the equation (1.5), which assures the global existence of the weak solutions of the initial-periodic boundary value problem for the equation (1.5). In Section 4, by the singular perturbation, we establish the existence theorem of the initial-periodic boundary value problem for the equation (1.1).