209. *A Note on C-compact Spaces*

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According to G. Viglino [7], a topological space $(X, \mathcal{T})$ is said to be *C-compact* if given a closed set A of X and a $\mathcal{T}$-open covering $\mathcal{U}$ of A, there is a finite number of elements of $\mathcal{U}$, say $U_i, 1 \leqq i \leqq n$, with $A \subset \bigcup_{i=1}^n \bar{U}_i$. It was shown by Viglino that in Hausdorff spaces the following implications hold and neither of them is reversible:

compact $\Rightarrow$ C-compact $\Rightarrow$ minimal Hausdorff.

Here a space X is *minimal Hausdorff* if X is Hausdorff and each open filter-base on X (i.e. a filter-base composed exclusively of open sets of X) with a unique adherent point is convergent.

The main results of this note are that (1) the product of a C-compact space and a compact space need not be C-compact in general, and that (2) there exist minimal Hausdorff spaces of arbitrary infinite cardinality which are not C-compact.

Theorem 1. *For any topological space X, the following properties of X are equivalent*:

(1) *X is C-compact,*

(2) *if A is a closed set of X and $\mathcal{F}$ a family of closed sets of X with $\cap \mathcal{F} \cap A = \emptyset$, then there is a finite number of elements of $\mathcal{F}$, say F_i, $1 \leqq i \leqq n$, with $\bigcap_{i=1}^n (\mathrm{Int}\, F_i) \cap A = \emptyset$.*

(3) *if A is a closed set of X and $\mathcal{G}$ an open filter-base on X whose elements have non-empty traces with A, then there is an adherent point of $\mathcal{G}$ in A.*

Proof. (1) $\Rightarrow$ (2). Let A be a closed subset of a C-compact space X and $\mathcal{F}$ a family of closed sets of X with $\cap \mathcal{F} \cap A = \emptyset$. Since $\mathcal{U} = \{X - F \mid F \in \mathcal{F}\}$ is a family of open sets of X covering A, there is a finite number of elements of $\mathcal{U}$, say $U_i = X - F_i$, $1 \leqq i \leqq n$, with $\bigcup_{i=1}^n \bar{U}_i \supset A$. Therefore, $\bigcap_{i=1}^n (\mathrm{Int}\, F_i) = X - \bigcup_{i=1}^n \bar{U}_i \subset X - A$.

(2) $\Rightarrow$ (3). Assume that there exist a closed set A and an open filter-base $\mathcal{G}$ on X having no adherent point in A whose elements have non-empty traces with A. Since $\mathcal{F} = \{\bar{G} \mid G \in \mathcal{G}\}$ is a family of closed sets of X with $\cap \mathcal{F} \cap A = \emptyset$, there is a finite number of elements of $\mathcal{F}$, say $F_i = \bar{G}_i$, $1 \leqq i \leqq n$, with $\bigcap_{i=1}^n (\mathrm{Int}\, F_i) \cap A = \emptyset$. Then we have $\bigcap_{i=1}^n G_i \cap A = \emptyset$. Since $\mathcal{G}$ is a filter-base, there is an element $G \in \mathcal{G}$ with $G \cap A = \emptyset$. This contradicts the assumption on $\mathcal{G}$.

(3) $\Rightarrow$ (1). Assume that X is not C-compact. There are a closed