230. *Characterization of Separable Polynomials over a Commutative Ring*

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Throughout this paper B will mean a commutative ring with an identity element, and all ring extensions of B will be assumed commutative with identity element coinciding with the identity element of B. Moreover, X will be an indeterminate, and by $B[X]$ denote the ring of polynomials in X with coefficients in B where $bX=Xb$ ($b \in B$). In [4], G. J. Janusz introduced the notion of separable polynomials over a commutative ring which is as follows: A polynomial $f(X) \in B[X]$ is called separable if it is a monic polynomial and if $B[X]/(f(X))$ is a separable B-algebra.[1] In [4, Theorem 2.2], it has been shown that under the assumption B has no proper idempotents, for a polynomial $f(X) \in B[X]$, $f(X)$ is separable if and only if there is a strongly separable B-algebra[2] A with no proper idempotents which contains elements $a_1, a_2, \cdots, a_n$ such that $f(X)=(X-a_1)(X-a_2)\cdots(X-a_n)$ and for $i \neq j$, $a_i - a_j$ is inversible in A. In [3], B. L. Elkins proved that if a polynomial $f(X) \in B[X]$ is separable then $f'(X+(f(X)))$ is an inversible element of $B[X]/(f(X))$, where $f'(X)$ is the derivative of $f(X)$. Recently, in [5], the present author proved that for a polynomial $f(X) \in B[X]$, if there is a ring extension of B which contains elements $a_1, \cdots, a_n$ such that $f(X)=(X-a_1)\cdots(X-a_n)$ and $\prod_{i \neq j}(a_i - a_j)$ is inversible in B then $f(X)$ is separable. The main purpose of this paper is to prove the following theorem.

Theorem 1. *Let $f(X) \in B[X]$. Then the following conditions are equivalent.*

(a) *$f(X)$ is separable.*

(b) *$f(X)$ is monic and $f'(X+(f(X)))$ is an inversible element of $B[X]/(f(X))$.*

(c) *There is a ring extension of B which contains elements $a_1, \cdots, a_n$ such that $f(X)=(X-a_1)\cdots(X-a_n)$ and $\prod_{i \neq j}(a_i - a_j)$ is inversible in B.*

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1) A commutative B-algebra S is called separable if it is a projective $(S \otimes_B S)$-module (cf. [1, p. 369]).

2) A B-algebra S is called strongly separable if it is finitely generated. projective, and separable over B.