18. *Mixed Problems of Hyperbolic Equations in a General Domain*

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§0. **Introduction.** Concerning to hyperbolic mixed problems with uniform Lopatinski's conditions, a priori estimates have been obtained by R. Sakamoto [5]. Our aim is to obtain existence theorems in general domains by making use of a priori estimates in a half space. To prove it, we make use of the method of partition of unity. So we must be sure two typical properties in a half spaces, that is, Huygens' principle and finite propagation speed. We sketch of the former in §1 and the latter in §2. Finally we state some results in general domains in §3.

Let us consider the problem $[A, \{B_i\}, \{D_t^i\}, f, \{g_j\}, \{u_j\}]$ in Ω_T;

$$(P)\begin{cases} A(D_x, D_t)u \equiv D_t^m u + \sum\limits_{\substack{|\nu|+j\leq m\\ j\leq m-1}} a_{\nu j} D_x^\nu D_t^j u = f \text{ in } \Omega_T = \Omega\times(0,T),\ (m=2m'),\\ B_i(D_x, D_t)u \equiv \sum\limits_{|\nu|+j=0}^{m_i} b_{\nu j}^i D_x^\nu D_t^j u = g_i \text{ on } S_T = S\times(0,T),\ (i=1,\cdots,m'),\\ D_t^j u = u_j \text{ on } \Omega_0\ (j=0,\cdots,m-1), \end{cases}$$

$\left(D_{x_j} = \dfrac{1}{i}\dfrac{\partial}{\partial x_j},\ D_x^\nu = D_{x_1}^{\nu_1}, \cdots, D_{x_n}^{\nu_n}, |\nu| = \nu_1 + \cdots + \nu_n\right.$ for multi index $\left.\nu = (\nu_1, \cdots, \nu_n)\right)$ where Ω is a domain in R^n whose boundary S is a C^∞ hypersurface. For $\{A, B_i\}$, we set following assumptions;

(1) $$a_{\nu_j} \in \mathscr{B}(\Omega_T), \qquad b_{\nu_j}^i \in \mathscr{B}(S_T).$$

(2) A is regularly hyperbolic with respect to t, that is, m roots in λ of the characteristic polynomial $A_0(\xi, \lambda) = 0$ are real for parameter $\xi \in R^n$ and

$$d = \inf_{\substack{(x,t)\in\Omega_T\\ \xi\in R^n, |\xi|=1\\ i\neq j}} |\lambda_i(x,t,\xi) - \lambda_j(x,t,\xi)| > 0. \quad (\lambda_1, \cdots, \lambda_m;\ \text{roots}).$$

In addition, A has the ellipticity with respect to the x-direction on S, that is,

$$|A_0(\xi, x)| \geqslant c|\xi|^m \text{ where } (x,t)\in S_T.$$

Let $N(x)$ and $T(x)$ be the unit inner normal and the tangent space to S at $x\in S$ respectively. From (2), $A_0(z) = A_0(\zeta + zN, \lambda) = 0$ has m' roots $z_1^+, \cdots, z_{m'}^+$ whose imaginary parts are positive for $(x,t)\in S_T$, $(\zeta, \lambda) \in T\times \boldsymbol{C}^1$, $\operatorname{Im}\lambda < 0$.