16. *Integration of Alexander-Spanier Cochains*

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The purpose of this note is to define the notion of integration on singular chains for Alexander-Spanier cochains and state some of its properties such as Stokes' theorem. The notions of the volume element with respect to a metric and integral operator with its symbol for a (compact) CW complex are also given. The Details will appear in the Journal of the Faculty of Science, Shinshu University, Vol. 5, 1970.

0. Alexander-Spanier cochains. For a topological space X, we set

$$\Delta_s(X)=\{(x, x, \cdots, x) \mid x \in X\} \subset X \times \overset{s+1}{\cdots} \times X.$$

We denote by $\mathfrak{R}$ a topological vector space over $\boldsymbol{R}$ or $\boldsymbol{C}$.

Definition. Two $\mathfrak{R}$-valued functions f and g on $U(\Delta_s(X))$, a neighborhood of $\Delta_s(X)$, are called equivalent if

$$f \mid V(\Delta_s(X)) = g \mid V(\Delta_s(X)),$$

for some neighborhood $V(\Delta_s(X))$ of $\Delta_s(X)$ and the equivalence class of f by this relation is called the germ of $f(at\ \Delta_s(X))$. The germ of f is denoted by $\bar{f}$ or simply, f.

Definition. A germ of f at $\Delta_s(X)$ is called an ($\mathfrak{R}$-valued) Alexander-Spanier s-cochain.

We call an Alexander-Spanier s-cochain $\bar{f}$ is continuous, regular or alternative if a representation f of $\bar{f}$ is continuous, $f(x_0, x_1, \cdots, x_s)=0$ if $x_i=x_j$ for some i, j or $f(x_{\sigma(0)}, x_{\sigma(1)}, \cdots, x_{\sigma(s)})=\operatorname{sgn}(\sigma) f(x_0, x_1, \cdots, x_s)$, $\sigma \in \mathfrak{S}^{s+1}$.

It is known that to define the coboundary operator δ by

$$\delta f(x_0, x_1, \cdots, x_{s+1})=\sum_{i=0}^{s+1}(-1)^i f(x_0, x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{s+1}),$$

we obtain

$$H^s(X, \mathfrak{R}) \simeq B^s(X, \mathfrak{R}) / Z^s(X, \mathfrak{R}),$$

if X is normal paracompact (cf. [1], [7]). Here $B^s(X, \mathfrak{R})$ and $Z^s(X, \mathfrak{R})$ are defined as usual for the group of Alexander-Spanier s-cochains (or continuous, regular or alternative s-cochains) and $H^s(X, \mathfrak{R})$ is the Čech cohomology group.

1. Definition of the integral. We use following notations:

$I^s=\{(t_1, \cdots, t_s) \mid 0 \leqq t_1 \leqq 1, \cdots, 0 \leqq t_s \leqq 1\}$,

$J=(j_1, \cdots, j_s)$, $j_1, \cdots, j_s$ are 0 or natural numbers,

$J+1_i=(j_1, \cdots, j_{i-1}, j_i+1, j_{i+1}, \cdots, j_s)$,