13. *Some Radii Associated with Polyharmonic Equations*

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Introduction. G. Pólya and G. Szegö [2] defined the inner radius of a bounded domain by a conformal correspondence from the domain to a disk and showed that it can be also given by the Green's function of the domain relative to the Laplace's equation $\Delta u=0$. In addition, they defined the biharmonic inner radius of a domain by the Green's function of the domain concerning the biharmonic equation $\Delta^2 u=0$. Using the results, they calculated the ordinary inner and biharmonic inner radii of a nearly circular domain. The aim of this paper is to extend the above results. In the first place, we obtain the Green's function of a disk relative to the n-harmonic equation $\Delta^n u=0$ and define the n-harmonic inner radius of a domain. On the base of the results, we compute the n-harmonic inner radius of a nearly circular domain and it is remarkable that it is monotonously decreasing with respect to integer n.

1. Inner radii associated with polyharmonic equations.

We use the following notations hereafter. Let D be a bounded domain, C the boundary of D, a an inner point of D, z the variable point in D and r the distance from a to z.

Definition 1. The function satisfying following two conditions is called the Green's function of D with the pole a relative to the n-harmonic equation $\Delta^n u=0$.

(1) The function has in a neighborhood of a the form

$$r^{2(n-1)}\log r+h_n(z),$$

where the function $h_n(z)$ satisfies the equation $\Delta^n u=0$ in D.

(2) On the boundary C, the function and all its normal derivatives of order $\leqq n-1$ vanish.

Theorem 1. *If D is the disk $|z|<R$ in the complex z-plane, the Green's function $G_n(a,z)$ of D with the pole a relative to the equation $\Delta^n u=0$ is as follows,*

$$G_n(a,z)=|z-a|^{2(n-1)}\log\left|\frac{R(z-a)}{R^2-\bar{a}z}\right|$$
$$-\frac{1}{2}\sum_{k=1}^{n-1}\frac{|z-a|^{2(n-k-1)}}{kR^{2k}}\{|R(z-a)|^2-|R^2-\bar{a}z|^2\}^k.$$

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