10. On Semilattices of Groups. II

By Sándor Lajos
K. Marx University of Economics, Budapest, Hungary

(Comm. by Kinjirô KUNUGI, M. J. A., Jan. 12, 1971)

This note is a continuation of, and is written in the same terminology as, the author's earlier paper [1]. There was proved the following result.

Theorem A. *A semigroup S is a semilattice of groups if and only if the intersection of any two bi-ideals of S equals to their product.*

This criterion has the following consequence.

Corollary. *Let S be a semilattice of groups. Then*

$$\bigcap_{i=1}^{k} B_i = \prod_{i=1}^{k} B_i \tag{1}$$

holds for any k bi-ideals $B_1, \cdots, B_k$ of S (k is an arbitrary fixed positive integer greater than one).

Here we show that property (1) is a necessary and sufficient condition for a semigroup S to be a semilattice of groups. First we prove this statement in case of $k=4$. The other cases can similarly be proved.

Theorem 1. *A semigroup S is a semilattice of groups if and only if the relation*

$$B_1 \cap B_2 \cap B_3 \cap B_4 = B_1 B_2 B_3 B_4 \tag{2}$$

holds for any four bi-ideals B_1, B_2, B_3, B_4 of S.

Proof. The necessity of the condition (2) is implied by the above Corollary of Theorem A.

Sufficiency. Let S be a semigroup with property (2) for every quadruplet of bi-ideals in S. Then (2) implies

$$B = SBS^2 \tag{3}$$

for every bi-ideal B of S, that is, every bi-ideal B of S is a two-sided ideal of S. To show that S is regular, let I be an arbitrary ideal of S. Then (2) implies

$$I = IS^2 I. \tag{4}$$

Hence we have $I \subset ISI$ and $ISI \subset I$, because I is a two-sided ideal of S. Consequently

$$I = ISI \tag{5}$$

for any ideal I of S. This implies that S is regular (cf. Luh [5]). Therefore S is a regular duo semigroup, i.e. S is a semilattice of groups (by Theorem 3 in author's paper [2]).

Theorem 2. *A semigroup S is a semilattice of groups if and only if the condition* (1) *holds for any k bi-ideals $B_1, \cdots, B_k$ of S (k is a fixed*