37. *Dynamical System with Ergodic Partitions*

By Yoshitaka YOKOI

(Comm. by Kinjirô KUNUGI, M. J. A., Feb. 12, 1971)

Introduction. In this paper we give a sufficient condition for a dynamical system on a compact metric Lebesque space to be ergodic. The following argument is essentially described in the work of Sinai [2]. The difference is the consideration of "measurable set" in place of "k-dimensional submanifold".

0. Notations. We denote by $(M, \mathfrak{B}, \mu)$ a Lebesque space with σ-algebra $\mathfrak{B}$ and a measure μ; $\mu(M)=I$. We suppose that M is a compact metric space with distance $d(\ ,\ )$.

(M, T, μ) is a dynamical system, i.e., T is an automorphism of $(M, \mathfrak{B}, \mu)$.

$\mathfrak{N}$ denotes the trivial subalgebra consisting of sets of measure zero or of measure one, and $\mathfrak{S}(A_\alpha)$ the σ-algebra generated by the system of measurable sets $\{A_\alpha\}$.

$\mathfrak{S}|_A$ means the restriction of a σ-subalgebra $\mathfrak{S}$ to a measurable set A.

1. Expansive partitions and contractive partitions.

Definition 1.1. Let $\xi=\{C_\xi\}$ be a partition of M into measurable sets $\{C_\xi\}$. ξ is called to be *T-expansive* (*T-contractive*), if for two points $x, y \in M$ which belong to the same element C_ξ of ξ $d(T^n x, T^n y)$ $(d(T^{-n}x, T^{-n}y))$ converges to zero as $n \to \infty$.

Theorem 1.1. *Let ξ, η be two partitions of M. If one is T-expansive and the other is T-contractive then any T-invariant summable function is $\mathfrak{S}(\xi) \cap \mathfrak{S}(\eta)$-measurable.*

2. The partition of M which is not necessarily measurable may be measurable, if it is considered locally in some sense.

Definition 2.1. $\mathfrak{U}=\{U_k | k=1, 2, \cdots\}$ is called *a local basis* of M, if 1) $\mathfrak{S}(\mathfrak{U})=\mathfrak{B}$, 2) for any measurable set A such that $0<\mu(A)<1$ there exists some $U_k \in \mathfrak{U}$ which satisfies;

$$\mu(A \cap U_k) \cdot \mu(A^c \cap U_k) \neq 0$$

Definition 2.2. Let $\mathfrak{U}=\{U_k | k=1, 2, \cdots\}$ be a local basis of M. $\{(U_k, \xi_k) | k=1, 2, \cdots\}$ is called a *measurable fibre structure* (m.f.s.), if

1) ξ_k is a measurable partition of U_k,

2) for almost all $x \in U_k \cap U_l$, $C_{\xi_k}(x) \cap U_l = C_{\xi_l}(x) \cap U_k$, where $C_{\xi_k}(x)$ is an element of ξ_k which contains x.

An m.f.s. $\{(U_k, \xi_k) | k=1, 2, \cdots\}$ defines an equivalence relation $\sim$ and hence induces a partition of M. The relation $x \sim y$ for $x, y \in M$ means that there exist $U_{k_1}, \cdots, U_{k_j} \in \mathfrak{U}$ such that $x \in U_{k_1}$, $y \in U_{k_j}$ and