35. *Surgery and Singularities in Codimension Two*

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1. Statement of results. Throughout this paper, W^{m+2} denotes a compact connected 1-connected PL $m+2$-manifold which is a Poincaré complex of formal dimension m. A closed PL submanifold L^m of W^{m+2} with codimension 2 is called a *homotopy spine* if the inclusion map $i: L^m \to W^{m+2}$ is a homotopy equivalence. In this paper, we shall formulate an obstruction theory to finding *locally flat* homotopy spines of W^{m+2}. The problem has been solved in odd dimensional case [1]. Here we shall consider the case where m is even: $m=2n \geqq 6$. An additional condition (H) on W^{2n+2} is also assumed, which is a generalization of simplicity condition for knots [3].

(H) *There exist an S^1-fibration $\xi \xrightarrow{p} W$ and a map $\phi: \partial W^{(n)} \to \xi$, where $\partial W^{(n)}$ is the n-skeleton of some triangulation of ∂W, such that* (i) *ϕ is n-connected and* (ii) *the diagram* $\partial W^{(n)} \xrightarrow{\phi} \xi$, $\partial W^{(n)} \to W \xleftarrow{p} \xi$ *is homotopically commutative.*

Note that $\pi_1(\partial W) \cong \pi_1(\xi)$ is a cyclic group. Denote this group in a multiplicative way by $J_q = \{t^m \mid m \in Z\}/(t^q)$, $q=0,1,2,\cdots$. In § 3, a covariant functor $P_{2n}(*)$ from the category {cyclic groups, onto homomorphisms} to the category {abelian groups, onto homomorphisms} is algebraically introduced. Our results are the following:

Theorem 1.1. *W^{2n+2} admits a locally flat homotopy spine if and only if a well defined obstruction element $\eta(W) \in P_{2n}(\pi_1 \partial W)$ is equal to zero.*

The groups $P_{2n}(J_q)$ have some interesting properties.

Theorem 1.2. (i) $P_{2n}(J_0) \cong C_{2n-1}$ *(Levine's knot cobordism group of $(2n-1, 2n+1)$-knots* [3]*), where J_0 is an infinite cyclic group.* (ii) $P_{2n}(1) \cong P_{2n}$*(Kervaire-Milnor's surgery obstruction group* [2]*), where 1 stands for a trivial group.* (iii) $P_{2n+4}(J_q) = P_{2n}(J_q)$.

A submanifold L^{2n} is said to be *1-flat* if it is locally flat except at a finite set of points. The obstruction $\eta(W)$ can be described in terms of singularities of 1-flat homotopy spines. We have proved in [1] that W^{2n+2} admits a 1-flat homotopy spine L^{2n}. Define the *singularity* at $p \in L$ by a $(2n-1, 2n+1)$-knot $\sigma_p(L) = (Lk(p, L), Lk(p, W))$. The *total singularity* of L^{2n} in W is the summation $\sigma(L) = \sum_{p \in L} \sigma_p(L)$ in Levine's