33. On Semigroups whose Ideals are All Globally Idempotent

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A left (right, two-sided) ideal X of a semigroup S is called globally idempotent if $X^2=X$ (according to S. Lajos). As is well-known, a commutative semigroup is regular if and only if every ideal is globally idempotent ([3]), and a normal semigroup is regular if and only if every left ideal is grobally idempotent ([5]). J. Calais characterized semigroups whose right ideals or left ideals are all globally idempotent ([2]). Recently R. C. Courter [2] has given an interesting characterisation of rings whose ideals are all globally idempotent. In this note we give a characterisation for a semigroup which is similar to Theorem 1.2 of R. C. Courter [2]. For another properties of semigroups whose ideals are all globally idempotent see [4] and [6].

Let X and Y be subsets of a semigroup S. Then we put

$$(Y:X)=\{s\in S: xs\in Y \quad \text{for all } x\in X\}$$
$$(Y:X)^*=\{s\in S: sx\in Y \quad \text{for all } x\in X\}.$$

Theorem 1. *The following statements on a semigroup S are equivalent:*

(1) *$X^2=X$ for every ideal X of S.*

(2) *$X\cap Y=XY$ for every ideals X and Y of S.*

(3) *$(Y:X)\cap X=X\cap Y$ for every ideals X and Y of S.*

(4) *$(Y:X)^*\cap X=X\cap Y$ for every ideals X and Y of S.*

(5) *$(Y:X)\cap Z=Y\cap Z$ for every ideals X, Y and Z of S such that $Z\subseteq X$.*

(6) *$(Y:X)^*\cap Z=Y\cap Z$ for every ideals X, Y and Z of S such that $Z\subseteq X$.*

(7) *$R\cap X\subseteq XR$ for every right ideal R and every ideal X of S.*

(8) *$L\cap X\subseteq LX$ for every left ideal L and every ideal X of S.*

(9) *$R\subseteq XR$ for every right ideal R and every ideal X of S such that $R\subseteq X$.*

(10) *$L\subseteq LX$ for every left ideal L and every ideal X of S such that $L\subseteq X$.*

(11) *$(R:X)\cap X\subseteq X\cap R$ for every right ideal R and every ideal X of S.*

(12) *$(L:X)^*\cap X\subseteq L\cap X$ for every left ideal L and every ideal X of S.*