30. On Dedekindian *l*-Semigroups and its Lattice-Ideals

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Our main purpose of the present note is to study some lattice-ideals of Dedekindian l-semigroups. The notation and terminology are those of [1].

1. Let S be an Artinian l-semigroup considered in [1]. An integral element q of S is called primary if the conditions $xy \leq q$, $x \not\leq q$ ($x, y \in I_G$) imply $y^\rho \leq q$ for some positive integer ρ. Then it can be proved that $p \equiv \sup\{x \in I_G \mid x^\rho \leq q$ for some positive integer $\rho\}$ is a prime element in I. Now let $\mathfrak{B}=\{v\}$ be a system of valuations with the properties (A), (B) and (C) in [1]. Then for any fixed $v \in \mathfrak{B}$ and for a primary element q with $q \leq p(v)$ (cf. [1; § 4]), we have that $v(q) \neq 0$ and $v'(q)=0$ for every $v' \in \mathfrak{B}$ with $v' \neq v$. By using the above fact and the results in [1; § 4], we can prove that, if p is a low prime element of I, the set of the minimal primes less than p consists of infinite many members.

Let p be prime and be not low in I. If we take a valuation $v \in \mathfrak{B}$ such that $v(p)>0$, then since $v(p) \geq v(p(v))=1$, we have $p(v) \geq p$. Now we suppose that $v(p(v))<v(p)$. Let z be an element such that $z<p$, $z \in I_G$ and $v(z)=v(p)$, and let u be an element such that $u \leq p(v)$, $u \in I_G$ and $v(u)=1$. Then we can take an element u' such as $zu^{-v(p)}u'=u_0 \leq e$ and $v(u_0)=0$. By using this and the property "$p(v_1) \neq p(v_2)$ for $v_1 \neq v_2$ in $\mathfrak{B}$", we can show that there exists one and only one valuation v such that $p(v)=p$, $v \in \mathfrak{B}$.

An Artinian l-semigroup is called *Dedekindian* if it has no low element different from e. Then we obtain that any Dedekindian l-semigroup S forms an l-group, and every element a of S is factored into a product of a finite number of primes $p(v)$: $a=\prod_{v \in \mathfrak{B}} p(v)^{v(a)}$, and the factorization is uniquely determined apart from its commutativity. In other words S is the restricted direct product of the cyclic groups $\{p(v)\}$, $v \in \mathfrak{B}$. Now let S be an Artinian l-semigroup. Then the following three conditions are equivalent:

(1) *S is Dedekindian.*

(2) *Each minimal prime of I is maximal.*

(3) *Any two distinct minimal primes are coprime.*

Ad (1)$\Rightarrow$(2): Let p be a minimal prime of I. Then p is written as $p=p(v)$ for some $v \in \mathfrak{B}$. Suppose that there exists an element a such as $p<a \leq e$. Then $v(p)>v(a)$ and $0 \leq v'(a) \leq v'(p)=0$ for $v' \neq v$, $v' \in \mathfrak{B}$. This