29. A Characterization of Artinian l-Semigroups

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The aim of the present note is to generalize Artin's well-known equivalence relation (quasi-equal relation) introduced in commutative rings for some sorts of commutative l-semigroups,[1] and to give a characterization of such l-semigroups by a system of valuations defined on some quotient l-semigroups with compactly generated cones.

1. Let S be a conditionally complete and commutative l-semigroup with unity quantity e, and let I be the cone (integral part) of S. We suppose throughout this paper that I is compactly generated by a compact generator system Σ containing e (cf. [7]), and that S is a quotient semi-group of I by Σ, that is, every element x of Σ is invertible in S and every element c of S can be written as $c=ax^{-1}$, where $a \in I$ and $x \in \Sigma$. If a compactly generated l-semigroup I with a compact generator system Σ is given, we can prove that there exists a quotient l-semigroup of I by Σ, if and only if the following two conditions hold for I and Σ: (i) for any two elements x and y of Σ, there exists an element a of I such that axy is in Σ, and (ii) every element of Σ satisfies the cancellation law. The lattice-structure is naturally introduced in the quotient semigroup, and such a quotient l-semigroup is uniquely determined within isomorphisms over I.

Now it can be proved that the join-semi-lattice generated by Σ is also a compact generator system of I. Hence we may assume, if necessary, that Σ is closed under finite join-operation. If, in particular, S forms a group, we can show that the maximal condition holds for the elements of I. By using this, we can prove the following: in order that a quotient l-semigroup of I by Σ is a group, it is necessary and sufficient that every element of I has a prime factorization and every prime is divisor-free in the sense of the partial-order.

2. In this and the next sections, we let S be a quotient l-semigroup (conditionally complete) of the cone I by a compact generator system Σ of I. The multiplicative group generated by Σ in S will be denoted by G. Then the element of S can be represented as a supremum of a subset of G. For any two elements a, b of S, the set $X_{a,b}$

1) Artin's equivalence relation has been introduced in various l-semigroups by many authors [1], [4], [2], [5], [3], etc. A systematic study was given in [4] and [5].