63. *On a Property of Behavior in Time for Solutions of the Wave Equation*

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In this note we treat the solutions of the wave equation with even space dimension. In the case of odd space dimension, it is easily verified that the solutions of the wave equation with initial data in $\mathcal{S}$ (=the totality of Schwartz's rapidly decreasing functions) decrease rapidly when t tends to infinity. On the other hand, in the case of even dimension, this is not always true. Generally the solutions can only decay with t^{-N}. For this reason we argue whether there are the solutions which decay rapidly when t tends to infinity. The similar problems for the solutions of the second order hyperbolic equations are treated by many authors. See [1], [2], [3], [4], [5].

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We consider the following Cauchy problem:

$$\frac{\partial^2 u}{\partial t^2}=\Delta u, \quad \begin{pmatrix}\Delta=n\text{-dimensional}\\ \text{Laplacian}; \ n=2p\end{pmatrix} \tag{1}$$

$$u(x,0)=\varphi(x)\in\mathcal{S}, \tag{2}$$

$$u_t(x,0)=\psi(x)\in\mathcal{S}. \tag{3}$$

As is well known, the above Cauchy problem has the following unique solution;

$$\begin{aligned} u(x,t)=&(2\pi)^{-p}\frac{d}{dt}\left(\frac{1}{t}\frac{d}{dt}\right)^{p-1}\left[t^{n-1}\int_{|\xi|\le 1}\frac{\varphi(x-t\xi)}{\sqrt{1-|\xi|^2}}d\xi\right]\\ &+(2\pi)^{-p}\left(\frac{1}{t}\frac{d}{dt}\right)^{p-1}\left[t^{n-1}\int_{|\xi|\le 1}\frac{\psi(x-t\xi)}{\sqrt{1-|\xi|^2}}d\xi\right].\end{aligned} \tag{4}$$

Theorem 1. *We fix an arbitrary x. For $u(x,t)$ decreases rapidly when t tends to infinity, it is necessary and sufficient that the following* (5) *and* (6) *are satisfied.*

$$\int|x-\xi|^{2m}\,\varphi(\xi)d\xi=0, \qquad (m=0,1,2,\cdots), \tag{5}$$

$$\int|x-\xi|^{2m}\,\psi(\xi)d\xi=0, \qquad (m=0,1,2,\cdots). \tag{6}$$

Proof. We put

$$J(t\,;\varphi)=t^{n-1}\int_{|\xi|\le 1}\frac{\varphi(x-t\xi)}{\sqrt{1-|\xi|^2}}d\xi=t^{-1}\int_{|\xi|\le 1}\frac{\varphi(x-\xi)}{\sqrt{1-|\xi|^2/t^2}}d\xi,$$