82. *On Integral Representation Involving Meijer's G-Function*

By R. S. DAHIYA
Department of Mathematics, Iowa State University

(Comm. by Kinjirô KUNUGI, M. J. A., April 12, 1971)

The object of the present paper is to study the following integral relation of Meijer's G-function:

$$\int_0^\infty\int_0^\infty \frac{y^{2u-1}x^{2v-1}(x^2+y^2)}{(a^2y^2+b^2x^2)^{u+v}} G_{p,q}^{m,n}\left(\frac{\alpha x^2y^2(x^2+y^2)}{(a^2y^2+b^2x^2)^2}\middle|\begin{matrix}a_p\\ b_q\end{matrix}\right) f(x^2+y^2)dxdy$$

$$(1)\qquad =\frac{\sqrt{\pi}\,2^{-1-u-v}}{a^{2u}b^{2v}}\int_0^\infty G_{p+2,q+2}^{m,n+2}\left(\frac{\alpha z}{4}\middle|\begin{matrix}1-u,1-v,a_p\\ b_q,1-\frac{u}{2}-\frac{v}{2},\frac{1}{2}-\frac{u}{2}-\frac{v}{2}\end{matrix}\right)f(z)dz,$$

where $|\arg\alpha|<(m+n-1/2\,p-1/2\,q)\pi, R(u,v)>0$; $f(z)=0(z^{-\delta})$ for large z and $f(z)=0(z^{\varepsilon-1/2})$ for small z; $\delta>0$, $\varepsilon>0$.

Meijer's G-function is defined [1] by a Mellin-Barnes type integral:

$$G_{p,q}^{m,n}\left(z\middle|\begin{matrix}a_p\\ b_q\end{matrix}\right)=G_{p,q}^{m,n}\left(z\middle|\begin{matrix}a_1,\cdots,a_p\\ b_1,\cdots,b_q\end{matrix}\right)$$

$$(2)\qquad =\frac{1}{2\pi i}\int_L \frac{\Gamma[(b_m)-s]\Gamma[1-(a_n)+s]}{\Gamma[1-(b_{m+1},q)+s]\Gamma[(a_{n+1},p)-s]}z^s\,dz,$$

where m,n,p,q are integers with $q\geq 1$; $0\leq n\leq p, 0\leq m\leq q$, the parameters a_j and b_j are such that no poles of $\Gamma(b_j-s), j=1,2,\cdots,m$ coincides with any pole of $\Gamma(1-a_j+s)$; $j=1,2,\cdots,n$. The poles of integrand must be simple and those of $\Gamma(b_j-s)$; $j=1,2,\cdots,m$ lie on one side of the contour L and those of $\Gamma(1-a_j+s)$; $j=1,2,\cdots,n$ must lie on the other side. The contour L runs from $-i^\infty$ to i^∞. Throughout the paper the above conditions shall be retained. The integral converges if $p+q<2(m+n)$ and $|\arg z|<(m+n-1/2\,p-1/2\,q)\pi$.

Then, by using the formula ([2], p. 377):

$$(3)\qquad \int_0^{\pi/2}\frac{\sin^{2u-1}\theta\cos^{2v-1}\theta}{(a^2\sin^2\theta+b^2\cos^2\theta)^{u+v}}d\theta=\frac{1}{2a^{2u}b^{2v}}\cdot\frac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)},\ R(u,v)>0,$$

we obtain our first result in the form

$$\int_0^{\pi/2}\frac{\sin^{2u-1}\theta\cos^{2v-1}\theta}{(a^2\sin^2\theta+b^2\cos^2\theta)^{u+v}}G_{p,q}^{m,n}\left(\frac{\alpha z\sin^2\theta\cos^2\theta}{(a^2\sin^2\theta+b^2\cos^2\theta)^2}\middle|\begin{matrix}a_p\\ b_q\end{matrix}\right)d\theta$$

$$(4)\qquad =\frac{\sqrt{\pi}\,2^{-u-v}}{a^{2u}b^{2v}}G_{p+2,q+2}^{m,n+2}\left(\frac{\alpha z}{4}\middle|\begin{matrix}1-u,1-v,a_p\\ b_q,1-\frac{u}{2}-\frac{v}{2},\frac{1}{2}-\frac{u}{2}-\frac{v}{2}\end{matrix}\right),$$

provided $R(u)>0, R(v)>0, p+q<2(m+n)$ and $|\arg z|<(m+n-1/2\,p-1/2\,q)\pi$.