77. The Theory of Nuclear Spaces Treated by the Method of Ranked Space. II

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In the paper [3], we defined the neighbourhood having a rank in the nuclear space Φ.

Now in this note we shall prove that the space Φ above is a linear ranked space.

§3. Definition of unit ball. Following §2, we suppose the mappings $T_{n_i}^{n_{i+1}}$, $i=0,1,2,\cdots$, in the nuclear space Φ. Furthermore, we consider a fixed sequence of real numbers $\{\varepsilon_i\}$ such that

(1) $$\varepsilon_1=1$$

(2) $$2\left(\sum_{k=1}^{\infty}\lambda_{k,n_i,n_{i+1}}\right)\varepsilon_{i+1}\leqq\varepsilon_i$$

(3) $$0<\varepsilon_{i+1}<\varepsilon_i.$$

Then we define $V_i(0,1,m)\equiv U_i(0,\varepsilon_i,m)$ as the unit ball of neighbourhood with rank i in regarding to m.

In particular, we define that the neighbourhood with rank 0, V_0, is always the space Φ.

By the definition of $U_i(0,\varepsilon_i,m)$ in §2, it is easily verified to be $rV_i(0,1,m)=V_i(0,r,m)$ for any $r>0$.

Then we shall call number r the radius of neighbourhood $V_i(0,r,m)$.

Lemma 5. *We have* $V_j(0,1,m)\supseteq V_i(0,1,m)$ *if* $j\leqq i$.

Proof. By Lemma 1, it is clear.

Lemma 6. *We have* $V_i(0,1,m')\supseteq V_i(0,1,m)$ *if* $m'\leqq m$.

Lemma 7. *We have* $V_i(0,r,m)\supseteq V_i(0,r'm)$ *if* $r'\leqq r$.

Now, we shall define the fundamental sequence of neighbourhoods.

Definition 1. When a sequence of neighbourhoods $\{V_{\gamma_i}(0,r_i,m_i)\}$ satisfies the following conditions, it is called the fundamental sequence.

(1) there exists some integer i_0 such that $V_{\gamma_i}(0,r_i,m_i)=V_0$ for $0\leqq i\leqq i_0$,

(2) $\gamma_i\leqq\gamma_{i+1}$ for $i>i_0$ and $\gamma_i\to\infty$,

(3) $r_i\geqq r_{i+1}$ for $i>i_0$ and $r_i\to 0$,

(4) $m_i\leqq m_{i+1}$ for $i>i_0$ and $m_i\to\infty$.

Lemma 8. *If* $\{V_{\gamma_i}(0,r_i,m_i)\}$ *is a fundamental sequence of neighbourhoods, then* $g\in V_{\gamma_i}(0,r_i,m_i)$ *for every* i *implies* $g=0$.

Proof. By Lemma 2, it is clear.

Lemma 9. (1) $V_i(0,r,m)$ *is circled.*