104. *A Remark on the Concept of Channels. III*

An Algebraic Theory of Extended Toeplitz Operators

By Marie CHODA and Masahiro NAKAMURA
Department of Mathematics, Osaka Kyoiku University

(Comm. by Kinjirô KUNUGI, M. J. A., May 12, 1971)

In the previous notes [1], a few elementary properties of generalized channels are discussed. In the present note, some problems on extended Toeplitz operators will be studied as a kind of generalized channels.

1. In the classical theory of Toeplitz operators, a *Laurent operator* l_ϕ on L^2 is defined by the multiplication by an essentially bounded function ϕ with functions of $L^2(\varphi \in L^2 \to \phi\varphi \in L^2)$ where L^2 is the Hilbert space of all square integrable functions defined on the unit circle with the normalized Lebesgue measure. A *Toeplitz operator* t_ϕ is defined by

(1) $$t_\varphi = pl_\phi \mid H^2,$$

where H^2 is the subspace of L^2 consisting those functions whose Fourier coefficients vanish on negative integers and where p is the projection belonging to H^2.

An abstraction of the above situation is recently given by Devinatz and Shinbrot [2]: An abstract Hilbert space H plays the role of L^2, and H^2 is replaced by an arbitrary (closed) subspace M. Every element a of $B(H)$, the algebra of all (bounded linear) operators, defines a *general Wiener-Hopf operator*

(1′) $$t_p(a) = pa \mid M,$$

where p is the projection belonging to M.

An another moderate abstraction is given by Douglas and Pearcy [4]. Every element of a maximally abelian von Neumann algebra A plays the role of Laurent operator. If each vector of M is separating in the sense of Dixmier [3] for A, M is called a *weak Riesz space.* If M and $M^\perp$ are weak Riesz subspaces for A, then M is called a *Riesz subspace.* A *Riesz system* is the triple (H, A, M). Every element $a \in A$ is called a *generalized Laurent operator* (simply (GL) *operator*) and $t_p(a)$ a *generalized Toeplitz operator* (simply (GT) *operator*).

2. Assume that A is a von Neumann algebra acting on H. Then the both cases are unified: $A = B(H)$ for the case of Devinatz-Shinbrot and A is maximally abelian for the case of Douglas-Pearcy. In the below, instead of $t_p(a)$, the following notation will be used:

(1″) $$a_p = pa \mid M.$$