97. On the Relation between the Positive Definite Quadratic Forms with the Same Representation Numbers

By Yoshiyuki KITAOKA
Nagoya University

(Comm. by Kenjiro SHODA, M. J. A., May 12, 1971)

1. In this note we investigate the relation between the positive definite integral quadratic forms with the same representation numbers.

2. A positive definite $n\times n$ matrix $A=(a_{ij})$ is called even positive if all a_{ij} are integers and all a_{ii} are even integers; then we put $\vartheta(\tau, A) = \sum_{\xi\in Z^n} e^{\pi i A[\xi]\tau}$.

For an even positive $2k\times 2k$ matrix A we define the level of A by the smallest natural number N such that NA^{-1} is also even positive; then N divides $\det A$ and $\det A$ divides N^{2k}.

An even positive ternary matrix $\begin{pmatrix} 2a & g & f \\ g & 2b & e \\ f & e & 2c \end{pmatrix}$, which is denoted by $[a, b, c, e, f, g]$ for brevity, is called reduced in the sense of Seeber and Eisenstein if the following conditions are satisfied:

1) e, f, g are all positive or all non-positive.

2) $a\leq b\leq c, a+b+e+f+g\geq 0$.

3) $|f|\leq a, |g|\leq a, |e|\leq b$.

4) If $a=b, |e|\leq|f|$; if $b=c, |f|\leq|g|$; if $a+b+e+f+g=0, 2a+2f+g\leq 0$.

5) For $e, f, g\leq 0$: if $a=-g, f=0$; if $a=-f, g=0$; if $b=-e, g=0$.

6) For $e, f, g>0$: if $a=g, f\leq 2e$; if $a=f, g\leq 2e$; if $b=e, g\leq 2f$.

We say that two matrices A, B are equivalent if $A={}^tTBT$ holds for some integral matrix T with determinant ± 1.

3. Theorem 1. *Assume that $\vartheta(\tau, A)=\vartheta(\tau, B)$ holds for two even positive matrices A, B. Then the following assertions* i), ii), iii) *and* iv) *are true.*

i) *There exists a matrix T with rational numbers as entries such that ${}^tTAT=B$ holds.*

ii) *In case that A is $2k\times 2k$ matrix, A and B belong to the same genus if the level N of A is* odd *or* $N\equiv 2 \bmod 4$.

iii) *In case that A is $(2k+1)\times(2k+1)$ matrix, A and B belong to the same genus if* $\det A=2^t r$ *holds, where $t\leq 4$ and r is odd.*

iv) *If A is $n\times n$ matrix with $n\leq 4$, A and B always belong to the same genus.*