119. A Path Space and the Propagation of Chaos for a Boltzmann's Gas Model

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In [3], we considered a model with infinite number of interacting particles. A gas model corresponding to a *spatially homogeneous Boltzmann equation with bounded scattering cross section* can be discussed in the frame work, but *not* the gas of hard spheres.[1]

Here, we construct a path space which describes, to some extent, the motion in the model, especially the way of interactions between particles. Next, we formulate a natural version of the *propagation of*

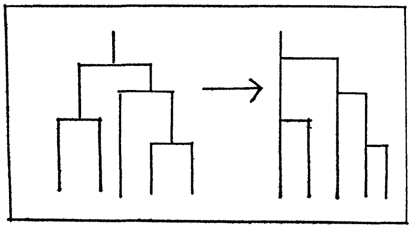

Fig. 1

chaos, which Kac [1] discovered. The version needs no approximation process with respect to the number of particles.

We use the notations and definitions in [3], but rewrite the figure of branches as in Fig. 1.[2]

1. Following two lemmas are fundamental.

Lemma 1. *For x in R^∞ and $s \leqslant t$,*

$$\sum_{b \in T} P(s, b(x), t, R) \leqslant 1. \tag{2}$$

In case $q(t, x)$ is bounded,

$$\sum_{b \in T} P(s, b(x), t, R) \equiv 1, \qquad x \in R^\infty. \tag{3}$$

Here, $b(x)$ for $x=(x_1, x_2, \cdots)$ in R^∞ denotes $b((x_1, x_2, \cdots, x_{\#(b)}))$.

Lemma 2. *For a branch b, $x=(x_1, \cdots x_{\#(b)})$, $E \in \mathscr{B}(R)$ and $s \leqslant t \leqslant u$,*

$$P(s, b(x), u, E) = \sum_{b' \leqslant b} \int_{R^{\#(b')}} \prod_{b_i \in b/b'} P(s, b_i(x_i), t, dy_i) P(t, b'(y), u, E). \tag{4}$$

1) The space in which the particles move here or in [3] is the velocity space in the original gas model. The reader can consult McKean [2] for a more realistic description and related problems.

2) We restrict ourselves to binary interactions as in II of [3].