116. *Modules over Bounded Dedekind Prime Rings. I*

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The purpose of this paper is to generalize the theory of modules over commutative Dedekind rings [3] to the case of modules over bounded Dedekind prime rings.

1. Definitions and notations. In this paper, all rings have identity and are associative, and modules are unitary. Ideals always mean two-sided ideals. Let R be a prime Goldie ring and let Q be the quotient ring of R. Then R is called a *Dedekind ring* if R is a maximal order in Q and every right (left) R-ideal is projective (see [8]). R is *bounded* if every integral one-sided R-ideal contains a non-zero ideal. Let M be an R-module. We say that $m \in M$ is a *torsion element* if there is a regular element c in R such that $mc=0$. Since R satisfies the Ore condition, the set of torsion elements of M is a submodule $T \subseteq M$. And M/T is evidently torsion-free (has no torsion elements). Let x be an element of M. Then we define $O(x)=\{r \in R \mid xr=0\}$ and say that $O(x)$ is an *order right ideal* of x. Let P be a prime ideal of R and let M be a torsion R-module. Then we say that M is *primary* (P-primary) if $O(x)$ contains a power of P for every element x in M. A submodule S of an R-module is said to be *pure* if $Sc=S \cap Mc$ for every regular element c in R. In particular, S is said to be *strongly pure* if $Sr=S \cap Mr$ for every element r in R. Then the following properties hold: (i) Any direct summand is strongly pure. (ii) A (strongly) pure submodule of a (strongly) pure submodule is (strongly) pure. (iii) The torsion submodule is pure. (iv) If M/S is torsion-free, then S is pure. We define an R-module M to be *divisible* if $Mc=M$ for all regular element c in R. Finally J or $J(R)$ always denotes the Jacobson radical of the ring R. The ring R is *local* if R/J is artinian and $\bigcap_{s=1}^{\infty} J^s=(0)$. R is *s-local* if R is local and R/J is a division ring.

2. Modules over bounded Dedekind prime rings. Let R be a semi-hereditary prime Goldie ring, let Q be the quotient ring of R and let M be a finitely generated torsion-free R-module. Then the sequence $0 \to M \to M \otimes_R Q$ is exact and $M \otimes_R Q$ is Q-projective. So $M \otimes_R Q$ is a submodule of a finitely generated free Q-module. Furthermore, since M is finitely generated, M is a submodule of a free R-module. Hence M is R-projective. Now let u be a uniform element of R. Then the short exact sequence $0 \to O(u) \to R \to uR \to 0$ splits. So R is a direct sum