136. *On Finite Groups whose Subgroups have Simple Core Factors**)

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If H is a subgroup of the finite group G then the core of H, denoted H_G, is $\bigcap_{x\in G} x^{-1}Hx$, the largest of the normal subgroups of G contained in H; the core factor of the subgroup H is H/H_G. G is called (see [1]) $\mathfrak{X}$-core (respectively, $\mathfrak{X}$-max-core) if all its subgroups (respectively, all its maximal subgroups) have core factors in the class $\mathfrak{X}$ of finite groups. $\mathfrak{X}$-max-core groups have been classified, for some $\mathfrak{X}$, in [1] and [4], but little is known about $\mathfrak{X}$-core groups. Of course, if $\mathfrak{X}=\{1\}$, $\mathfrak{X}$-core groups are precisely the Hamiltonian groups; the purpose of this paper is to give information about $\mathfrak{X}$-core groups close to Hamiltonian groups-that is, groups whose core factors are relatively uncomplicated.

Throughout this paper, all groups considered are finite. Unless otherwise specified, references and notation are drawn from [3]. Let $\mathfrak{S}, \mathfrak{C}$, and $\mathfrak{P}$ denote the classes of all groups which are simple, cyclic, and of prime-power order, respectively (including the trivial group). Then $\mathfrak{S}\cap\mathfrak{C}=\mathfrak{S}\cap\mathfrak{P}$ is the class of all groups of order a prime. We begin by showing that $\mathfrak{S}$-core groups are $\mathfrak{S}\cap\mathfrak{C}$-core groups.

(1) **Theorem.** *$\mathfrak{S}$-core groups are solvable.*

Proof. We recall from [1] that subgroups and homomorphic images of $\mathfrak{S}$-core groups are again $\mathfrak{S}$-core groups. Now let G be a minimal counterexample. If $1\neq N\lhd G, N\neq G$, then N and G/N are $\mathfrak{S}$-core groups and by induction must be solvable, making G solvable, a contradiction. Therefore G is simple. But then all subgroups of G have trivial core and hence must be simple too-even the Sylow subgroups. This means G has only cyclic Sylow subgroups and so by a theorem of Hölder (p. 420, [3]) is solvable.

(2) **Corollary.** *If G is an $\mathfrak{S}$-core groups and $H\leq G$ then H_G has index at most a prime in H.*

(3) **Corollary.** *If G is an $\mathfrak{S}$-core group then $F(G)$, the Fitting subgroup, has index at most a prime in G.*

Proof. This follows directly from Proposition (7) of [1].

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