170. *On Some Subgroups of the Group Sp(2n, 2)*

By Eiichi BANNAI*)
University of Tokyo

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Introduction. We say that a subgroup H of a group G is of rank 2, if the number of double cosets $H\backslash G/H$ is equal to 2. Any subgroup of rank 2 of G is the stabilizer of a point of some doubly transitive permutation representation of G, and vice versa. It is known that the symplectic group $Sp(2n, 2)$ has two kinds of subgroups of rank 2 of index $2^{n-1}(2^n+1)$ and $2^{n-1}(2^n-1)$ which are isomorphic to the groups $0(2n, 2, +1)$ and $0(2n, 2, -1)$, respectively. Here $0(2n, 2, +1)$ and $0(2n, 2, -1)$ denote the orthogonal group of index n and $n-1$ defined over a field with 2 elements, respectively.

The purpose of this note is to give an outline of the proof of the following Theorem 1 which asserts that the two kinds of subgroups mentioned above are the only subgroups of rank 2 of the group $Sp(2n,2)$. Details will be published elsewhere.

Theorem 1. *Let H be a subgroup of rank 2 of the group $Sp(2n, 2)$, $n\geqslant 3$. Then either*

1) *H is of index $2^{n-1}(2^n+1)$ and is isomorphic to the group $0(2n, 2, +1)$, or*

2) *H is of index $2^{n-1}(2^n-1)$ and is isomorphic to the group $0(2n, 2, -1)$.*

§1. The group $Sp(2n, 2)$.

We may define $G=Sp(2n, 2)$, the symplectic group defined over the finite field $GF(2)$, by

$$G=\left\{X \in GL(2n, 2)\,;\, {}^tXJX=J, \text{ with } J=\begin{pmatrix} & I_n \\ I_n & \end{pmatrix}\right\}.$$

Here I_n denotes the $n\times n$ identity matrix, and the unwritten places of any matrix always represent 0. The group $G=Sp(2n, 2)$ is simple if $n\geqslant 3$.

Let us define some subgroups of the group G as follows:

$$Q=\left\{X \in GL(2n, 2)\,;\, X=\begin{pmatrix} I_n & B \\ & I_n \end{pmatrix}, \text{ with } {}^tB=B\right\},$$

$$L=\left\{X \in GL(2n, 2)\,;\, X=\begin{pmatrix} A & \\ & {}^tA^{-1} \end{pmatrix}, \text{ with } A \in GL(n, 2)\right\},$$

$$R=\left\{X \in GL(2n, 2)\,;\, X=\begin{pmatrix} A & \\ & {}^tA^{-1} \end{pmatrix}, \text{ where } A \text{ is any upper triangular unipotent } n\times n \text{ matrix}\right\}.$$

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