168. *Freely Generable Classes of Structures*

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A class K of structures is said to be freely generable, if for any (non-empty) set E of generator symbols and any set Ω of defining relations, there exists a freely generated structure in K presented by E and Ω. The conditions for a class of algebras to be freely generable were studied in [1; § 8 in Chap. III] and [2]. Our main purpose of this note is to show a new necessary and sufficient condition for a class of structures to be freely generable.

A structure $\mathfrak{A}$ of the similarity type corresponding to a first order language L is simply called a structure for L. The domain of $\mathfrak{A}$ is denoted by $D[\mathfrak{A}]$. A formula Φ of L which contains at most some of $x_1, \cdots, x_n$ as free variables is denoted by $\Phi(x_1, \cdots, x_n)$ if the free variables $x_1, \cdots, x_n$ need to be indicated. Let $\Phi(x_1, \cdots, x_n)$ be any formula of L, and let $a_1, \cdots, a_n$ be elements in $D[\mathfrak{A}]$. Then we write $(\mathfrak{A}; a_1, \cdots, a_n) \models \Phi(x_1, \cdots, x_n)$, if $a_1, \cdots, a_n$ satisfy $\Phi(x_1, \cdots, x_n)$ in $\mathfrak{A}$ when the free variables $x_1, \cdots, x_n$ are assigned the values $a_1, \cdots, a_n$ respectively. An atomic formula of L means a formula of the form $t_1 = t_2$ or of the form $r(t_1, \cdots, t_m)$, where r is an m-ary relation symbol of L and $t_1, \cdots, t_m$ are terms of L. Let $\mathfrak{A}$ and $\mathfrak{B}$ be structures for a first order language L. A mapping h of $D[\mathfrak{A}]$ onto (or into) $D[\mathfrak{B}]$ is called an L-homomorphism of $\mathfrak{A}$ onto (or into) $\mathfrak{B}$, if for any atomic formula $\Theta(x_1, \cdots, x_n)$ of L and for any elements $a_1, \cdots, a_n$ in $D[\mathfrak{A}]$, $(\mathfrak{A}; a_1, \cdots, a_n) \models \Theta(x_1, \cdots, x_n)$ implies $(\mathfrak{B}; h(a_1), \cdots, h(a_n)) \models \Theta(x_1, \cdots, x_n)$. An L-homomorphism h of $\mathfrak{A}$ onto $\mathfrak{B}$ is called an L-isomorphism of $\mathfrak{A}$ onto $\mathfrak{B}$ if the mapping h is one to one and the inverse mapping h^{-1} is also an L-homomorphism. Let E be a set of constant symbols (i.e. nullary operation symbols) not belonging to L. Then, a new first order language can be obtained from L by adjoining all the constant symbols $e \in E$, which is denoted by $L(E)$. If $L(E)$ contains at least one constant symbol, then E is said to be L-generative. Now let $\mathfrak{A}$ be a structure for L, and ψ a mapping of E into $D[\mathfrak{A}]$. Then $\mathfrak{A}$ can be expanded to a structure for $L(E)$, by considering $\psi(e)$ as interpretations of e in $\mathfrak{A}$, and the expanded structure is denoted by $\mathfrak{A}(\psi)$.

Let K be a class of structures for L. Let E be a set of constant symbols not belonging to L, and Ω a set of atomic sentences (i.e. atomic formulas without free variables) of $L(E)$. Now let $\mathfrak{A}$ be a structure