8. On Uniqueness and Estimations for Solutions of Modified Frankl' Problem for Linear and Nonlinear Equations of Mixed Type

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1. Introduction. Concerning the Frankl' problem for equations of mixed type which has been proposed by F. I. Frankl' (see [3] and references quoted there), the uniqueness and the maximum principles are discussed under some restrictions by many authors in USSR using the method of singular integral equations or the abc method. In the present paper, with the intention of using Agmon, Nirenberg and Protter type maximum principle [1] we slightly modify the boundary condition of the problem on some hyperbolic boundary in such a way as the directional derivative of the solution in the direction of a characteristic is given, while in the Frankl's original problem the derivative with respect to x is given. Then we prove the uniqueness and lead some estimations for the solutions of linear and nonlinear problems. On the basis of the above modification, we infer that we shall be able to return to the discussion of the original problem, e.g. the uniqueness of the solution and the existence of a weak solution, but these matters will be discussed elsewhere.

2. Definitions and problems. Let $K(y)$ be a function of y defined and twice continuously differentiable on an interval $(-y_1, y_2)$ where y_1, $y_2>0$, and which has the property $yK(y)>0$ for $y\neq 0$.

We shall define a domain Ω in the x, y-plane satisfying the condition that the ordinates of the points of the closure $\bar{\Omega}$ are contained in the interval $(-y_1, y_2)$ as follows. Let us take two points $A(a, 0)$ and $B(b, 0)$ on the x-axis with $a<b$ and let C be the intersection point of two curves in $y<0$, one issuing from A has the slope $0\geq dx/dy> -\sqrt{-K(y)}$ and the other issuing from B has the slope $0\leq dx/dy<\sqrt{K(y)}$. We shall denote the arcs AC and BC by γ_1 and γ_2, respectively. Further, let $D(d, 0)$ and $E(e, 0)$ be two points on the x-axis with $d<a, e>b$ and let σ be a Jordan arc in $y>0$ joining D and E where it is assumed that the length of σ is not less than the length l of γ_1. Let F be a point on σ such that the length of the arc DF denoted by σ_0 equals l. Ω shall be the domain enclosed with the curve $ACBEFDA$. Let $\Omega_1=\Omega\cap\{y>0\}$ and $\Omega_2=\Omega\cap\{y<0\}$.