6. *Perfect Class of Spaces*

By Keiô NAGAMI
Department of Mathematics, Ehime University

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The author introduced in [6] the notion of perfect class of spaces and showed that the class of ν-spaces is perfect. Recall that a class $\mathfrak{C}$ of spaces is said to be perfect if the following five conditions are satisfied.

(1) If $X \in \mathfrak{C}$, then X is normal.

(2) If $X \in \mathfrak{C}$ and $Y \subset X$, then $Y \in \mathfrak{C}$.

(3) If $X_i \in \mathfrak{C}$, $i=1, 2, \cdots$, then $\Pi X_i \in \mathfrak{C}$.

(4) If $X \in \mathfrak{C}$, then there exists $Z \in \mathfrak{C}$ with $\dim Z \leqq 0$ such that X is the image of Z under a perfect mapping.

(5) If $X \in \mathfrak{C}$ and Y is the image of X under a perfect mapping, then $Y \in \mathfrak{C}$.

It is to be noted that the first three conditions imply that each element of $\mathfrak{C}$ is perfectly normal. The aim of this paper is to show the existence of the maximal perfect subclass in the class of paracompact σ-spaces. A characterization theorem of dimension of cubic μ-spaces will also be stated. All spaces in this paper are assumed to be Hausdorff and all mappings to be continuous. The suffix i runs through the positive integers. Definitions for undefined terminologies can be seen in [6]. The discussion with Professor K. Morita at Shuzenji Hot Spring Symposium, 1970, was suggestive to the present study.

Lemma 1. *If X is a paracompact Σ-space with* $\dim X=0$ *and Y is a paracompact Morita space with* $\dim Y=0$, *then* $\dim (X\times Y)=0$.

This can be proved by almost the same way as in the proof of [3, Theorem 3].

Lemma 2 ([1, Theorem 4]). *Let X be the inverse limit of $\{X_i, \pi^i{}_j\}$, where each X_i is a normal space with* $\dim X_i \leqq n$ *and each $\pi^i{}_j$ is open. If X is countably paracompact, then X is a normal space with* $\dim X \leqq n$.

Lemma 3. *Let X_i, $i=1, 2, \cdots$, be paracompact Σ-spaces with* $\dim X_i=0$. *Then* $\dim (\prod X_i)=0$.

Proof. Since a Σ-space is a Morita space by [2, Theorem 2.7], $\dim (X_1\times X_2)=0$ by Lemma 1. Let $\prod_{i\leqq j} X_i$, $j>2$, be an arbitrary finite product. Since $\prod_{i<j} X_i$ is a paracompact Σ-space by [2, Theorem 3.13], we can prove easily $\dim (\prod_{i\leqq j} X_i)=0$ by induction with the aid of Lemma 1. Since the infinite product $\prod X_i$ is paracompact by [2, Theorem 3.13], then $\dim (\prod X_i)=0$ by Lemma 2. The proof is finished.