# 73. *On the Existence of Quasiperiodic Solutions of Nonlinear Hyperbolic Partial Differential Equations*

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## 1. Introduction.

In this note we shall consider a global property, that is, the quasiperiodic property, of the solutions of the following quasilinear one dimensional wave equation with dissipative term $\alpha u_t$, where $\alpha$ is a constant:

(1) $$M(u)=u_{tt}-u_{xx}+\alpha u_t=h(x, t, u, u_x, u_t),$$

where $h$ is quasiperiodic with basic frequencies $\omega_1, \cdots, \omega_m$ in $t$. We shall show the existence of such quasiperiodic solutions of the form (1) that have the same basic frequencies as $h$ and satisfy the boundary conditions $u(0, t)=u(\pi, t)=0$. These solutions are classical solutions.

The case $m=1$ is the periodic case and was already solved by Rabinowitz [1], [2]. Especially, in [2] equation is strictly nonlinear.

## 2. Notations and definitions.

**Definition.** $f(x, t)$ is called *quasiperiodic* with basic frequencies $\omega_1, \cdots, \omega_m$ in $t$, if there exists a function $F(x, \theta_1, \cdots, \theta_m)$ such that $f(x, t)=F(x, \omega_1 t, \cdots, \omega_m t)$, where $F(x, \theta_1, \cdots, \theta_m)$ is a continuous function of period $2\pi$ in $\theta_1, \cdots, \theta_m$. Basic frequencies $\omega_1, \cdots, \omega_m$ are real numbers. We shall denote by $\mathscr{B}^k(\omega_1, \cdots, \omega_m)$ the class of $f(x, t)$ for which $\mathscr{F}(x, \theta_1, \cdots, \theta_m)$ is $C^k$-class in $x, \theta_1, \cdots \theta_m$ and by $\mathscr{F}^k(\omega_1, \cdots, \omega_m) \subset \mathscr{B}^k(\omega_1, \cdots, \omega_m)$ the class of $f(x, t)$ which is $2\pi$-periodic in $x (1 \leqq k \leqq \infty)$. Every $f(x, t) \in \mathscr{F}^k$ is expanded in the Fourier series if $k \geqq 1$:

$$f(x, t)=\sum_{j\in \mathbf{Z}, k\in \mathbf{Z}^m} f_{jk} e^{ijx} e^{i(\omega, k)t}.$$

We introduce the norms in $F^k$ by $\|f\|=\sum |f_{jk}|$ and

$$\|f\|_1=\|f\|+\|f_x\|+\|f_t\|.$$

Now we assume that $h(x, t, p, q, r)$ is in the form

$$f(x, t)+g(x, t, p, q, r), f(x, t) \fallingdotseq 0.$$

Then we can represent $g(x, t, p, q, r)$ in the form $G(x, \omega_1 t, \cdots, \omega_m t, p, q, r)$, where $G(x, \theta_1, \cdots, \theta_m, p, q, r)$ is continuous and $2\pi$-periodic in $\theta_1, \cdots, \theta_m$. Further we assume that $f(x, t)$ and $g(x, t, u, u_x, u_t)$ vanish at the boundary $x=0, x=\pi$.

## 3. The existence of quasiperiodic solutions.

**3.1.** At first we consider the case where the forcing term $h(x, t, u, u_x, u_t)$ does not depend on $u, u_x, u_t$:

(2) $$M(u)=u_{tt}-u_{xx}+\alpha u_t=f(x, t).$$