67. *Spinnable Structures on Differentiable Manifolds*

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(Comm. by Kôsaku YOSIDA, M. J. A., May 12, 1972)

In this note we shall introduce a new structure, called a spinnable structure, on a differentiable manifold. Roughly speaking, a differentiable manifold is spinnable if it can spin around an axis as if the top spins. Such structure originated in connection with the study of foliations (Lawson [1], Tamura [4], [5]), and the existence of a codimension-one foliation follows from a spinnable structure of a special kind.

Definition 1. An m-dimensional differentiable manifold M^m is called *spinnable* if there exists an $(m-2)$-dimensional submanifold X satisfying the following conditions:

(i) The normal bundle of X is trivial.

(ii) Let $X\times D^2$ be a tubular neighborhood of X, then $C=M^m-X\times \operatorname{Int} D^2$ is the total space of a fibre bundle ξ over a circle.

(iii) Let $p: C\to S^1$ be the projection of ξ, then the diagram

$$\begin{tikzcd} X\times S^1 \arrow[r, "\iota"] \arrow[dr, "p'"'] & C \arrow[d, "p"] \\ & S^1 \end{tikzcd}$$

commutes, where ι denotes the inclusion map and p' denotes the natural projection onto the second factor.

The submanifold X is called an *axis* and a fibre F of ξ is called a *generator*. F is an $(m-1)$-dimensional submanifold of M^m. Obviously $\partial F=X$ holds if $\partial M^m=\emptyset$. The fibre bundle $\xi=\{C, p, S^1, F\}$ is called a *spinning bundle* and the pair (X, ξ) is called a *spinnable structure*[1)] of M^m.

Example 1. S^m is spinnable with (naturally imbedded) S^{m-2} as axis. $\boldsymbol{R}^m$ is spinnable with $\boldsymbol{R}^{m-2}$ as axis.

Remark 1. It is easy to see that, if M^m is spinnable, then the decomposition $M=(F\times I)\cup(F\times I)$ holds.

The following theorem is easily proved.

Theorem 1. *Suppose that M_1^m and M_2^m are spinnable connected differentiable manifolds having connected submanifolds X_1 and X_2 as*

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1) The same concept is called a fibered knot in a recent paper of A. Durfee and H. Lawson, Fibered knots and foliations of highly connected manifolds.