83. *An Approach to Quasiclassical Approximation for the Schrödinger Equation*

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(Comm. by Kôsaku YOSIDA, M. J. A., June 2, 1972)

§1. **Introduction.** The purpose of the present paper is to show by a certain new approach that there is a geometric- and wave-optical relation, in the literal sense of the word, between the classical mechanics and the quantum mechanics by discussing, as an example and also as an application of our approach, the problem of quasiclassical approximation for the Schrödinger equation. Considerations about the notion of characteristics for the Schrödinger equation lead us to our approach. Our method is to introduce a new real variable s and to look upon the equation in a space of dimension larger than that of the original space by 1 and to apply techniques of the geometric optics to the transformed Schrödinger equation which turns out to be a strongly hyperbolic equation of the second order. Our new variable s has a physical meaning as the action of a motion and leads us to a new formulation of the classical mechanics of particles to which the classical Hamilton-Jacobi theory is reduced.

The present paper is a brief summary of the methods and some results. The proofs and a detailed version of some parts of the present paper will be published in a forthcoming paper.

§2. **Transformation of the Schrödinger equation.** Let us start with the consideration of Maslov's proposal concerning the notion of characteristics for the Schrödinger equation. Maslov [7] asked of what type the Schrödinger equation is and proposed to take as a characteristic equation for the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Phi=\frac{1}{2\mu}\sum_{1}^{n}\left(-i\hbar\frac{\partial}{\partial x_j}\right)^2\Phi+V(x,t)\Phi,\ \Phi=\Phi(x,t,\hbar) \tag{2–1}$$

its corresponding Hamilton-Jacobi equation

$$\frac{\partial S}{\partial t}+\frac{1}{2\mu}\sum_{1}^{n}\left(\frac{\partial S}{\partial x_j}\right)^2+V(x,t)=0. \tag{2–2}$$

This equation plays important roles in the W.K.B. expansion for the Schrödinger equation. However, this does not seem to be a natural choice of the characteristic equation of the Schrödinger equation for the following reasons. First, for the Klein-Gordon equation, for