80. A Remark on the Asymptotic Behavior of the Solution of $\dddot{x}+f(\ddot{x})\ddot{x}+\phi(\dot{x},\ddot{x})+g(\dot{x})+h(x)=p(t,x,\dot{x},\ddot{x},\dddot{x})$

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1. Introduction. This paper is concerned with the equation of the form

(1.1) $$\ddddot{x}+f(\dddot{x})\dddot{x}+\phi(\dot{x},\ddot{x})+g(\dot{x})+h(x)=p(t,x,\dot{x},\ddot{x},\dddot{x})$$

where f,ϕ,g,h and p are continuous real-valued functions depending only on the arguments shown, and the dots indicate differentiation with respect to the independent variable t.

We shall investigate conditions under which all solutions of (1.1) tend to zero as $t\to\infty$. Much work has been done on the asymptotic properties of non-linear differential equations of the fourth order and many of these conditions are summerized in [7, Kapitel 6].

M. Harrow [5] established conditions under which every solution of the equation

(1.2) $$\ddddot{x}+a\dddot{x}+f(\ddot{x})+g(\dot{x})+h(x)=p(t) \qquad (p(t)\equiv 0)$$

tends to zero as $t\to\infty$. A. S. C. Sinha and R. G. Hoft [8] also considered the asymptotic stability of the zero solution of the equation

(1.3) $$\ddddot{x}+f(\dddot{x})\dddot{x}+\phi(\dot{x},\ddot{x})\ddot{x}+\psi(\dot{x})+\theta(x)=p(t) \qquad (p(t)\equiv 0).$$

In [1], M. A. Asmussen studied the behavior as $t\to\infty$ of the solution of the equation

(1.4) $$\ddddot{x}+f(\dddot{x})\dddot{x}+a_2\ddot{x}+g(\dot{x})+h(x)=p(t,x,\dot{x},\ddot{x},\dddot{x})$$

where a_2 is a positive constant. In this note the same conclusion for the more general equation (1.1) are obtained under the conditions slightly weaker than those of [5], [8] and [1].

2. Assumptions and Theorem. Throughout this paper we shall make the following assumptions:

(I) the function $f(z)$ is continuous in R^1,

(II) the functions $\phi(y,z)$ and $\dfrac{\partial\phi}{\partial y}(y,z)$ are continuous in R^2,

(III) $g(y)$ is a C^1-function in R^1,

(IV) $h(x)$ is a C^1-function in R^1,

(V) the function $p(t,x,y,z,w)$ is continuous in $[0,\infty)\times R^4$.

Henceforth the following notations are used;

$$g_1(y)=\frac{g(y)}{y}\quad (y\neq 0), \qquad g_1(0)=g'(0),$$