103. The Asymptotic Formulas for Eigenvalues of Elliptic Operators which Degenerate at the Boundary

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1. Introduction and Theorem.

The purpose of this paper is to derive asymptotic formulas with remainder estimates for the distribution of eigenvalues of elliptic operators which degenerate on the boundary. The formula without remainder estimate was established by I. A. Solomešč [6]. We investigate our problem under assumptions similar to those of [6]. Only the theorem and an outline of its proof are presented here and the details will be published elsewhere.

Let Ω be a bounded domain of $\boldsymbol{R}^n$ having the restricted cone property ([1]). Let $\{O_i\}$ and $\{C_i\}$ be the covering of $\partial\Omega$ and the set of corresponding cones, respectively, as guaranteed by the restricted cone property. Let ξ be any unit vector which is a positive multiple of the vector in the cone C_i. Then we assume that there is a constant K_0 such that

$$\delta(x_0+t\xi)\geq K_0\{\delta(x_0)+t\}$$

for any $x_0\in O_i$ and $0\leq t\leq h_i$, where h_i is the height of C_i and $\delta(x)=\text{dist}\,(x,\partial\Omega)$.

The set of complex-valued functions $f\in C^{m*}(\Omega)$ having a finite integral

$$\|u\|^2_{m,a}=\int_\Omega \delta(x)^a\sum_{|\alpha|\leq m}|D^\alpha f|^2dx$$

is denoted by $C^{m*}(\Omega)$ and the closure of $C^{m*}(\Omega)$ and $C_0^\infty(\Omega)$ with respect to the norm $\|\ \|_{m,a}$ is denoted by $W_{m,a}(\Omega)$ and $\mathring{W}_{m,a}(\Omega)$ respectively. Let V be some closed subspace of $W_{m,a}(\Omega)$ containing $\mathring{W}_{m,a}(\Omega)$ and B be an integro-differential sesquilinear form of order m

$$B[u,v]=\int_\Omega\sum_{|\alpha|=|\beta|=m}a_{\alpha\beta}(x)D^\alpha u\overline{D^\beta v}\,dx+B_1[u,v]$$

satisfying

$$\text{Re}\,B[u,u]\geq\delta\|u\|^2_m\qquad\text{for any } u\in V,$$

and

$$|B_1[u,v]|\leq K_1\{\|u\|_{m,a}\|v\|_{m-1,a}+\|u\|_{m-1,a}\|v\|_{m,a}\}\qquad\text{for } u,v\in V$$

where δ and K_1 are some positive constants. For the coefficients we shall assume that they are symmetric (i.e. $a_{\alpha\beta}(x)=\overline{a_{\beta\alpha}(x)}$), belong to $C^\infty(\Omega)$ and there is a positive constant K_2 such that