100. On Surfaces of Class VII_0

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1. In this short note we consider the surfaces satisfying the following conditions:

$(*)$ $\quad b_1=1, b_2=0$; the surfaces contain no curves.

We give two kinds of examples satisfying $(*)$, and give a theorem which determines the surfaces satisfying $(*)$ under an additional assumption. As a result of this theorem, we give three corollaries. The first of the corollaries is proved independently by Enrico Bombieri by a similar method.

Details will be published elsewhere.

2. Let $M \in SL(3, \boldsymbol{Z})$ be a unimodular matrix, with one real and two non-real eigenvalues, $\alpha, \beta, \bar{\beta}$, where $\alpha\beta\bar{\beta}=1$ and $\alpha>1$. Let $\boldsymbol{a}=\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ be a real eigenvector of α and $\boldsymbol{b}=\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ an eigenvector of β. Let G_M be the group generated by the analytic automorphisms:

$$(W, Z) \to (W+m_1a_1+m_2a_2+m_3a_3, Z+m_1b_1+m_2b_2+m_3b_3),$$
$$(m_1, m_2, m_3) \in \boldsymbol{Z}^3,$$
$$(W, Z) \to (\alpha W, \beta Z),$$

of $\boldsymbol{H} \times \boldsymbol{C}$, where $\boldsymbol{H}$ is the upper half-plane. The action of G_M on $\boldsymbol{H} \times \boldsymbol{C}$ is properly discontinuous and fixed point free. Now we define an analytic surface S_M to be $\boldsymbol{H} \times \boldsymbol{C}/G_M$. Then S_M is differentiably a 3-torus bundle over a circle, $b_1(S_M)=1$, $b_2(S_M)=0$, and S_M has the following properties.

Proposition 1.

i) *S_M contains no curves,*

ii) $\dim H^0(S_M, \Theta)=\dim H^1(S_M, \Theta)=\dim H^2(S_M, \Theta)=0$.

3. Let $N=(n_{ij}) \in SL(2, \boldsymbol{Z})$ be a unimodular matrix with two real eigenvalues, $\alpha, 1/\alpha$, where $\alpha>1$. Let

$$\boldsymbol{c}=\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{a}=\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

be real eigenvectors of α and $1/\alpha$, respectively. We fix an arbitrary complex number t and fix two integers, p, q, such that

$$0 \leq p, \ q \leq |\det(N-I)|-1.$$

Let $\boldsymbol{b}=\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ be the solution of the following equation: