152. A Treatment of Some Function Spaces used for the Study of Hypoellipticity. I

By Hideo YAMAGATA

Department of Mathematics, College of Engineering, University of Osaka Prefecture, Mozu, Sakai City, Osaka

(Comm. by Kinjirô KUNUGI, M. J. A., Nov. 13, 1972)

Introduction. The space $\mathfrak{F}(\Omega)\equiv\bigcap_{i\in I}B^{\mathrm{loc}}_{p_i,k_i}(\Omega)$ according to L. Hörmander [1] p. 45, p. 77 rather shows a common structure of the spaces belonging to a family. Then we will show here the above structure (with the extended form) described in the form of ranked space ([2] p. 4) in Theorem I-2 etc., § 1, and show the concrete meaning of transcendental ranks appearing in our ranked space in Example I-1, § 1. Next, we will show the concrete spaces as the special form of "the space in § 1" in Theorems I-3, I-4, § 2. Our extension is based on the unified description of the theorems on hypoellipticity which is related to C^∞ and related to a set of analytic functions defined in [3] p. 820 (cf. [1] p. 102, p. 178). The contents of this paper is a part of our further aim "the constructive systematization (i.e. ranked systematization by using transcendental ranks) for the theory of partial differential equation", because ranked space has a sort of totally ordered structure defined by the inclusion of pre-neighbourhoods with larger ranks.

§ 1. Extension of $\mathfrak{F}(\Omega)$ as a ranked space. Hereafter, we use the following notations; $K\equiv\{k(\xi)\,;\,0\leqslant k(\xi+\eta)\leqslant(1+C|\xi|)^N k(\eta)$, where $C, N>0$, $\xi,\eta\in R^n\}$, $B_{p,k}\equiv\left\{u\,;\,u\in(\mathfrak{D}'),\ \hat{u}\equiv\mathfrak{F}u\to\text{a function},\ \|u\|_{p,k}\equiv\left((2\pi)^{-n}\int|k(\xi)\hat{u}(\xi)|^p\,d\xi\right)^{1/p}<+\infty\right\}$, where $k\in K$, $1\leqslant p\leqslant\infty$ and $\|u\|_{\infty,k}\equiv\operatorname{ess\,sup}|k(\xi)\hat{u}(\xi)|$. $B^*_{p,k}\equiv\left\{u\,;\,u\in(\mathfrak{D}'),\ \mathfrak{F}^{-1}(k(\xi)\hat{u}(\xi))\to\text{a function},\ \|u\|^*_{p,k}\equiv\left((2\pi)^{-n}\int|\mathfrak{F}^{-1}(k(\xi)\hat{u}(\xi))|^{p'}\,d\xi\right)^{1/p'}<+\infty\right\}$, where $p'=p/(p-1)$, $p'=1$ for $p=\infty$, and $p'=\infty$ for $p=1$. Ω; open connected set in R^n. $L(\Omega)\subseteq\{f\,;\,\text{Carrier } f\subset\Omega\}$. P; diff. op. etc., $B^{\mathrm{loc}}_{p,k}(\Omega\,;L,P)\equiv\{u\,;\,Pu\in(\mathfrak{D}'_\Omega),\ \varphi Pu\in B_{p,k}$ for $^\forall\varphi\in L(\Omega)\}$, $B^{\mathrm{loc}}_{p,k}(\Omega\,;L,P)\equiv\{u\,;\,Pu\in(\mathfrak{D}'_\Omega),\ \varphi Pu\in B^*_{p,k}$ for $^\forall\varphi\in L(\Omega)\}$, $B^{\mathrm{loc}}_{p,k}{}^*(\Omega)\equiv B^{\mathrm{loc}}_{p,k}(\Omega\,;C_0^\infty,1)$. If $B^{\mathrm{loc}}_{p,k}(\Omega\,;L,P)=B^{\mathrm{loc}}_{p,k}(\Omega\,;\tilde{L},P)$ or $B^{\mathrm{loc}}_{p,k}{}^*(\Omega\,;L,P)=B^{\mathrm{loc}}_{p,k}{}^*(\Omega\,;\tilde{L},P)$, we say that these spaces (in the left hand side) are countably local, where $\tilde{L}(\Omega)=$countable subset of $L(\Omega)$. There exists $\tilde{C}_0^\infty(\Omega)$ for $C_0^\infty(\Omega)$ (cf. [1] p. 44).

Definition I-1. Let I be a totally ordered set of limit or isolated