163. *Regularity of Solutions of Hyperbolic Mixed Problems with Characteristic Boundary*

By Mikio TSUJI

(Comm. by Kinjirô KUNUGI, M. J. A., Dec. 12, 1972)

§1. Introduction. At first we recall the following well-known property of a solution of a hyperbolic Cauchy problem which is L^2-well posed: If the initial value is in $H^r(R^n)$, then the solution is also in $H^r(R^n)$ for any time >0. We call this "The property of having finite r-norm is persistent".

The author proved in [2] that, for a mixed problem to a first order hyperbolic system, if this mixed problem is L^2-well posed and the boundary is not characteristic for the equation, then the property of having finite r-norm is persistent.

In this note we discuss whether the persistent property holds or not in the case where the boundary is characteristic for the equation. Let Ω be a sufficiently smooth domain in R^n, $M=\partial/\partial t-L(t,x;D_x)$ be a first order hyperbolic system whose coefficients are $N\times N$ matrices in $\mathcal{B}([0,T]\times\Omega)$ and $P(t,x)$ be an $N\times N$ matrix defined on $[0,T]\times\partial\Omega$. Let us consider the mixed problem

$$(\mathrm{P})\begin{cases}(1.1) & M[u(t,x)]=f(t,x) & \text{in } [0,T]\times\Omega\\ (1.2) & u(0,x)=\varphi(x) & \text{on } \Omega\\ (1.3) & P(t,x)u(t,x)=0 & \text{on } [0,T]\times\partial\Omega.\end{cases}$$

Definition. The mixed problem (P) is said to be L^2-well posed if for any initial data $\varphi(x)\in D_0=\{u(x)\in H^1(\Omega)\,;\,P(0,x)u|_{\partial\Omega}=0\}$ and any second member $f(t,x)\in\mathcal{E}_t^0(H^1(\Omega))\cap\mathcal{E}_t^1(L^2(\Omega))$[1] there exists a unique solution $u(t,x)$ of (P) in $\mathcal{E}_t^1(L^2(\Omega))\cap\mathcal{E}_t^0(\mathcal{D}(L(t)))$ satisfying the following energy inequality

$$(1.4)\qquad \|u(t)\|\leqq c(T)\left(\|\varphi\|+\int_0^t\|f(s)\|\,ds\right),\qquad t\in[0,T],$$

where $c(T)$ is a positive constant which depends only on T.

We remark that $\mathcal{D}(L(t))$ is the closure of $D_t=\{u(x)\in H^1(\Omega)\,;\,P(t)u|_{\partial\Omega}=0\}$ by the norm $\|u\|_{L(t)}=\|u\|+\|L(t)u\|$. At first we state

Theorem 1. *In the case where* $\Omega=R_+^2=\{(x,y)\,;\,x>0,y\in R^1\}$, $L=\begin{bmatrix}-2 & 0\\ 0 & 0\end{bmatrix}\partial/\partial x+\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\partial/\partial y$ *and* $P=[1\quad 0]$, *the mixed problem* (P) *is L^2-well posed, but the property of having finite r-norm is not persistent. More precisely, if the initial value* $\varphi(x,y)\in H^m(R_+^2)$ *satisfies*

1) $\mathcal{E}_t^k(E)$ is the set of E-valued functions of t which are k-times continuously differentiable.