6. A Remark on Fluid Flows through Porous Media

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(Comm. by Kôsaku YOSIDA, M. J. A., Jan. 12, 1973)

1. Introduction. According to Muskat [10], the mathematical model for flow through a homogeneous porous medium is the following degenerate quasilinear parabolic equation

$$\frac{\partial u}{\partial t}=\Delta u^m, \tag{$*$}$$

where u is the density distribution, Δ is the Laplace-Beltrami operator in the space variable x and m is a real constant >1. Physically $m-1$ is the ratio of specific heats c_p/c_v. Equations of this type are of great importance in technology (see Ames [1], 1.2); besides they have some properties which seem interesting from a purely mathematical point of view. See Oleinik *et al.* [11], § 4; the author would refer the reader to the recent elaborate studies by Aronson [2]-[4]. (Of course the equation ($*$) has been studied by many other authors from various interesting aspects.)

To avoid unnecessary technical difficulties we concentrate our attention to flows through a medium which occupies all of the circle S^1. We consider the following Cauchy problem

$$\begin{cases} \dfrac{\partial u}{\partial t}=\dfrac{\partial^2}{\partial x^2}u^m & \text{in } S^1\times(0,T), \\ u|_{t=0}=a(x) & x\in S^1, \end{cases} \tag{1}$$

where a is a given non-negative function on S^1 called a initial datum and $0<T<\infty$. If $a\in C(S^1)^+$ and $da^m/dx\in L^\infty(S^1)$, then the Cauchy problem (1) has a unique "weak solution" u such that $u\in C(S^1\times[0,T])^+$ and $\partial u^m/\partial x\in L^\infty(S^1\times(0,T))$ (cf. Oleinik *et al.* [11]). Here and throughout the paper we use the usual vector lattice notation, i.e., $C(S^1)^+$ is the cone of all non-negative elements of $C(S^1)$ etc. da^m/dx and $\partial u^m/\partial x$ are distribution derivatives of $a^m\in\mathcal{D}'(S^1)$ and $u^m\in\mathcal{D}'(S^1\times(0,T))$ respectively.

The purpose of the present paper is to show the *continuous dependence of weak solutions on the initial data* in the sense of $L^1(S^1)$. Our result reads:

Theorem. *Suppose that*

$$a,\hat{a}\in C(S^1)^+ \quad \textit{and} \quad \frac{d}{dx}a^m,\frac{d}{dx}\hat{a}^m\in L^\infty(S^1).$$

Let u and $\hat{u}$ be the corresponding unique weak solutions of (1) *such that*