94. *Codimension 1 Foliations on Simply Connected 5-Manifolds*

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1. Recently N. A'Campo [1] has shown that every simply connected, closed 5-manifold with vanishing second Stiefel-Whitney class admits a codimension 1 foliation. The essential point in his construction is to utilize Smale's classification theorem [4].

In this note, similarly utilizing Barden's result [2], we show that every simply connected, closed 5-manifold admits a codimension 1 foliation. All the manifolds and the foliations considered here, are smooth of class C^∞.

2. Preliminaries. a) The second Stiefel-Whitney class $\omega^2(M)$ of a simply connected manifold M may be regarded as a homomorphism $\omega^2: H_2(M:Z)\to Z_2$, and we may consider ω^2 to be non-zero on at most one element of a basis. In a simply connected 5-manifold, the value of ω^2 on the homology class carried by an imbedded 2-sphere is the obstruction to the triviality of its normal bundle. Such a "non-zero valued" class has order 2^i for some positive integer i. Then i is a diffeomorphism invariant $i(M)$ of M.

D. Barden [2] has classified simply connected, closed, smooth 5-manifolds under diffeomorphism. Such a manifold is determined by $H_2(\ )$ and $i(\ )$. More precisely:

Proposition 1 [2]. *Simply connected, closed, smooth, oriented 5-manifolds are classified under diffeomorphism as follows. A canonical set of representatives is* $\{X_j \# M_{k_1} \# \cdots \# M_{k_s}\}$, *where* $-1\leqq j\leqq\infty$, $s\geqq 0$, $1<k_1$ *and* k_i *divides* k_{i+1} *or* $k_{i+1}=\infty$. *A complete set of invariants is provided by* $H_2(M)$ *and* $i(M)$. (for the notation, see [2], p. 373.)

b) S^2-bundles over S^2 with group SO_3 are classified by $\pi_1(SO_3)\cong Z_2$. We denote by A the product, and by B the non-trivial bundle. Next consider reductions of the structure group to SO_2, which are classified by $\pi_1(SO_2)\cong Z$. Let T_k be the S^2-bundle associated with the reduction given by the integer k. Furthermore, let x be the class in $H_2(T_k)$ of the sphere imbedded as the cross-section, corresponding to the "south pole", and y be the class of the sphere imbedded as a fiber. If $\cdot$ denotes the intersection number of homology class, then $x\cdot x=k$, $x\cdot y=1$ (we have the orientation of y to ensure this) and $y\cdot y=0$. For the homology bases of A, B, we shall reduce the bundles as T_0, T_1. Then we have, in [5]