81. *On Deformations of Quintic Surfaces*

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Let S_0 be a non-singular hypersurface of degree 5 in the projective 3-space $\boldsymbol{P}^3$ defined over $\boldsymbol{C}$. For brevity, we call S_0 a non-singular quintic surface.

By a surface, we shall mean a compact complex manifold of complex dimension 2, unless explicit indications are given. We say that a surface S is a deformation of S_0 if there exists a finite sequence of surfaces $S_0, S_1, \cdots, S_k, \cdots, S_n=S$ such that, for each k, S_k and S_{k-1} belong to one and the same complex analytic family of surfaces.

If S is a deformation of a non-singular quintic surface, S has the following numerical characters:

(*) $$p_g=4, \quad q=0, \quad c_1^2=5,$$

where p_g, q and c_1^2 denote the geometric genus, the irregularity and the Chern number of S, respectively. In particular S is an algebraic surface (see [5], Theorem 9). Moreover, since S_0 is minimal, Theorem 23 of Kodaira [5] asserts that

(**) S is minimal.

In this note, we shall give a statement of the results on the structures and deformations of surfaces which satisfy the conditions (*) and (**). Details will be published elsewhere.

1. Structures.

Theorem 1. *Let S be a minimal algebraic surface with $p_g=4$, $q=0$, and $c_1^2=5$. Then the canonical system $|K|$ on S has at most one base point. There are two cases:*

Case I. *$|K|$ has no base point. In this case, there exists a birational holomorphic map $f: S \to S'$ of S onto a (possibly singular) quintic surface S' in $\boldsymbol{P}^3$. S' has at most rational double points as its singularities.*

Case II. *$|K|$ has one base point b. Let $\pi: \tilde{S} \to S$ be the quadric transformation with center at b. Then this case is divided as follows:*

Case IIa. *There exists a surjective holomorphic map $f: \tilde{S} \to \boldsymbol{P}^1 \times \boldsymbol{P}^1$ of degree 2.*

Case IIb. *There exists a surjective holomorphic map $f: \tilde{S} \to \sum_2$ of degree 2, where $\sum_2$ denotes the Hirzebruch surface of degree 2, i.e., $\sum_2$ is a rational ruled surface with a section Δ_0 with $(\Delta_0)^2=-2$.*

The proof is based on a detailed analysis of the rational map Φ_K: