156. *On the Elementary Partitions of the State Set in a Multiple-Input Semiautomaton*

By Masami ITO
Kyoto Sangyo University, Kyoto

(Comm. by Kinjirô KUNUGI, M. J. A., Nov. 12, 1973)

1. Introduction. Determination of all homomorphic images of a given semiautomaton is equivalent to constructing all admissible partitions of its state set.

For the case of a one-input semiautomaton, there exists an efficient method for the construction of all admissible partitions. This can be done easily by determining all elementary partitions [1], [2].

For the case of a multiple-input semiautomaton, it seems complicated at first sight. But, even in this case, if all elementary partitions can be constructed, we can use the same procedure as the one-input case and we can obtain all admissible partitions.

In this note, we shall give an algorithm for constructing all elementary partitions of the state set in a multiple-input semiautomaton by using known elementary partitions for the one-input case. We shall borrow many notations and terms from [1].

2. Preliminaries. Consider a semiautomaton $A=(S, \Sigma, M)$, where S is a set of states, $\Sigma=\{\sigma_1, \sigma_2, \cdots, \sigma_n\}$ $(n\geq 2)$ is a set of inputs, and M is a set of transition mappings.

Definition 1. Let π be a partition of S. $\tilde{\pi}$ is called the admissible closure of π in A if and only if $\tilde{\pi}=\Pi_{i\in\Lambda}\xi_i$, where $\{\xi_i\,;\, i\in\Lambda\}$ is the set of all admissible partitions in A such that $\pi\leq\xi_i(i\in\Lambda)$.

In section 4, we shall give a method for constructing the admissible closure $\tilde{\pi}$ of π.

Definition 2. An admissible partition $\pi\neq 0$ of S in A, where 0 means the identity partition, is called elementary if and only if for every admissible partition π' of S in A, $0\leq\pi'\leq\pi$ implies $\pi'=0$ or $\pi'=\pi$.

3. Structure of elementary partitions. For the semiautomaton given in the preceding section, we shall construct following one-input semiautomata:

Put $\Sigma_i=\{\sigma_i\}$ and $M_i=\{\sigma_i^A\}=\{\sigma_i^{A_i}\}$ for each natural number i $(1\leq i\leq n)$. Thus, we obtain the one-input semiautomata $A_i=(S, \Sigma_i, M_i)$ $(1\leq i\leq n)$.

For each semiautomaton A_i $(1\leq i\leq n)$, the set of all elementary partitions of S in A_i can be determined by the procedure introduced in [1], [2]. We denote this set by $\mathcal{P}_i$.