146. *Average Powers of Gaussian White Noise*

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§ 1. Introduction and Theorem.

The purpose of this note is to prove a limit theorem of the average powers of the gaussian white noise, which is a generalization of the Brownian oscillation due to P. Lévy [3 ; § 41].

P. Lévy developed extensively Gâteaux's constructive study of the gaussian white noise in his book [2]. In his original idea the choice of the complete orthonormal system $\{\xi_n\}$ in the real Hilbert space $L^2[0,1]$ plays important roles, however there seems to be no other result depending on the choice of the system $\{\xi_n\}$. Our theorem stated below does depend on the choice of the system $\{\xi_n\}$.

Now, we shall introduce the measure of gaussian white noise. Let E be a nuclear subspace of the real Hilbert space $L^2[0,1]$ which is dense in the space $L^2[0,1]$ and satisfies the relation

$$E \subset L^2[0,1] \subset E^*,$$

where E^* stands for the dual space of E. For the characteristic functional

$$C(\xi)=\exp\left(-\frac{1}{2}\|\xi\|^2\right),$$

$\|\xi\|$ being the $L^2[0,1]$-norm of $\xi \in E$, there corresponds a probability measure μ on E^* such that,

$$C(\xi)=\int_{E^*} e^{i(x,\xi)}\mu(dx),$$

where (x,ξ), $x \in E^*$, $\xi \in E$ is the canonical bilinear form which links the spaces E and E^* We call μ the measure of gaussian white noise (see T. Hida [1]).

Next we define *the average powers* $\{\rho_N(x) ; x \in E^*\}_{N=1}^{\infty}$ for a complete orthonormal system $\{\xi_n\}_{n=1}^{\infty}$ in $L^2[0,1]$ as follows;

$$\rho_N(x)=\frac{1}{N}\sum_{n=1}^{N}(x, \xi\cdot\xi_n{}^2),$$

where ξ is a bounded measurable function on the interval $[0,1]$.

The system $\{\xi_n\}_{n=1}^{\infty}$ is called *weakly equally dense*, if it satisfies

$$\lim_{N\to\infty}\int_0^1 \eta(u)\left(\frac{1}{N}\sum_{n=1}^{N}\xi_n^2(u)-1\right)du=0,$$

for any bounded measurable function η on $[0,1]$.

Theorem. *Let* $\{\xi_n\}_{n=1}^{\infty}$ *be a complete orthonormal system. Then*