26. *On Bounded Reinhardt Domains*

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§ 1. Let D be a connected bounded Reinhardt domain in n-complex Euclidean space $\boldsymbol{C}^n$ which contains the origin o, and Aut (D) be the group of all biholomorphic transformations of D onto itself. The identity component of Aut (D) is denoted as $\mathrm{Aut}^0(D)$ $(=G)$. (For any bounded domain, it is a classical theorem of H. Cartan that $\mathrm{Aut}^0(D)$ becomes a Lie transformation group of D in a natural manner.)

When $n=2$, P. Thullen [5] determined the bounded Reinhardt domains with the property such that $G\cdot o \supsetneqq \{o\}$, ($G\cdot o$ denotes the G-orbit of the origin). In fact, such domains are holomorphically equivalent to the polydisc $\{(z,w)\in \boldsymbol{C}^2\,;\,|z|<1, |w|<1\}$, or to the *Thullen domain* $\{(z,w)\in \boldsymbol{C}^2\,;\,|z|^2+|w|^\alpha<1\ (\alpha>0)\}$. Recently, I. Naruki [4] and M. Ise [2] have treated a class of Reinhardt domains containing the higher-dimensional generalization of Thullen domains.

In this note, we intend to generalize these works and, further, to classify bounded Reinhardt domains in the n-dimensional case from the group theoretic point of view. The full exposition will be given elsewhere. The author is gratefull to Prof. Mikio Ise for suggesting the present problem and for his advices.

§ 2. Throughout this note, D will represent a bounded Reinhardt domain in $\boldsymbol{C}^n, \mathfrak{g}(D)$ the Lie algebra of complete holomorphic vector fields, and $\mathfrak{k}(D)$ the subalgebra of $\mathfrak{g}(D)$ which consists of all elements vanishing at the origin. Then, $\mathfrak{g}(D)$ can be identified canonically with the Lie algebra of G $(=\mathrm{Aut}^0(D))$ where $\mathfrak{k}(D)$ corresponds to that of the isotropy subgroup K of G with respect to the origin.

Since D is a circular domain, K consists of linear transformations of $\boldsymbol{C}^n$ and a transformation defined by $k_\theta : z \to e^{i\theta}z$ $(\theta \in \boldsymbol{R}, z\in \boldsymbol{C}^n)$ belongs to the center of K.

Now, we can write the vector field X contained in $\mathfrak{g}(D)$ in the form:

$$X=\sum p^k(\partial/\partial z^k),$$

where $z^1,\cdots,z^n$ denote the coordinates in $\boldsymbol{C}^n$, and p^k $(k=1,\cdots,n)$ holomorphic functions on D. A vector field X is said to be a polynomial vector field if the components p^k of X are polynomials of $z^1,\cdots,z^n$ and X is homogeneous of degree λ, if each p^k is a homogeneous polynomial of degree λ. For example, a vector field defined by $\partial=\sum \sqrt{-1}z^k(\partial/\partial z^k)$