47. *Complex Hypersurfaces with Vanishing Bochner Curvature Tensor*

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1. Introduction. The purpose of this paper is to show the following:

Theorem. *Let M be a complex hypersurface of complex dimension n $(n\geqq 2)$ in a space of constant holomorphic sectional curvature $\tilde{c}$. If the Bochner curvature tensor of M vanishes identically, then M is of constant holomorphic sectional curvature $\tilde{c}$.*

2. Preliminaries. Let $(\tilde{M}, J, g)$ be a Kaehlerian manifold of constant holomorphic sectional curvature $\tilde{c}$ of complex dimension $n+1$ $(n\geqq 2)$. Then the curvature tensor $\tilde{R}$ of $\tilde{M}$ is given by

$$\tilde{R}(\tilde{X},\tilde{Y})\tilde{W}=\frac{\tilde{c}}{4}\{g(\tilde{Y},\tilde{W})\tilde{X}-g(\tilde{X},\tilde{W})\tilde{Y}+g(J\tilde{Y},\tilde{W})J\tilde{X} -g(J\tilde{X},\tilde{W})J\tilde{Y}+2g(\tilde{X},J\tilde{Y})J\tilde{W}\}, \tag{1}$$

where $\tilde{X}$, $\tilde{Y}$ and $\tilde{W}$ are vector fields on $\tilde{M}$. Let M be a complex hypersurface of $\tilde{M}$ immersed by $\varphi: M\to\tilde{M}$ and ξ a local field of unit vectors normal to M. Then, identifying, for each $x\in M$, the tangent space $T_x(M)$ with $\varphi_*(T_x(M))\subset T_{\varphi(x)}(\tilde{M})$ by means of φ_*, we may put

$$\tilde{\nabla}_X\xi=-AX+s(X)J\xi, \tag{2}$$

$$\tilde{\nabla}_XY=\nabla_XY+h(X,Y)\xi+k(X,Y)J\xi, \tag{3}$$

where $\tilde{\nabla}$ denotes the covariant differentiation with respect to g, X and Y are vector fields in M and $-AX$ (resp. ∇_XY) is the tangential part of $\tilde{\nabla}_X\xi$ (resp. $\tilde{\nabla}_XY$). It is well known that the naturally induced metric is the Kaehlerian metric and the almost complex structure is the Kaehlerian structure on M. We denote them also by g and J respectively. Then the relations $h(X,Y)=g(AX,Y)$, $k(X,Y)=g(JAX,Y)$ and $JA=-AJ$ hold (for details, see [3]).

The curvature tensor R and the Ricci tensor S of M are given by

$$R(X,Y)=\tilde{R}(X,Y)+AX\wedge AY+JAX\wedge JAY, \tag{4}$$

$$S(X,Y)=-2g(A^2X,Y)+\frac{n+1}{2}\tilde{c}g(X,Y), \tag{5}$$

where $X\wedge Y$ is the endomorphism defined by $(X\wedge Y)(Z)=g(Z,Y)X-g(X,Z)Y$. The Bochner curvature tensor B of M is, by definition, given by