63. On the Homotopy Groups of Spheres

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(Comm. by Kôsaku YOSIDA, M. J. A., April 18, 1974)

The present note is concerned with the 2-component of the homotopy groups of spheres. Let π_*^n be the 2-component of the homotopy group $\pi_*(S^n)$. The groups π_{n+i}^n for $i\leq 22$ and all n have been determined in [6], [8], [9]. (If n is large, π_{n+i}^n is the 2-component of the i-th stable homotopy group of sphere spectrum and many data have been obtained by making use of the Adams spectral sequence.) In this note we are mainly concerned with the case of small n, namely unstable range.

§ 1. π_{n+i}^n for $i=23$ and 24.

The first purpose of this note is to announce the results on π_{n+i}^n for $i=23$ and 24. We completely determine the group structure of π_{n+23}^n and π_{n+24}^n for all n, by constructing the generators of π_*^n geometrically. Our method is the so-called composition method established by Toda [9]. The basic tool is the $EH\Delta$-exact sequence

$$(1.1)\qquad \cdots\longrightarrow \pi_{i+2}^{2n+1}\xrightarrow{\Delta}\pi_i^n\xrightarrow{E}\pi_{i+1}^{n+1}\xrightarrow{H}\pi_{i+1}^{2n+1}\xrightarrow{\Delta}\pi_{i-1}^n\longrightarrow\cdots$$

introduced by Whitehead and James, where E is the suspension homomorphism, H is the Hopf homomorphism and Δ is essentially the Whitehead product $[\iota_n,\ ]$. This enables us to calculate π_*^n inductively.

We now summarize the results of our calculation in the following theorem. The detailed calculations will be given in the forthcoming paper [7].

Theorem 1.2.***)

π_{n+23}^n and π_{n+24}^n are given by the table below.

(a) $\pi_{25}^2=\{\eta_2\circ\varepsilon_3\circ\kappa_{11}\}\approx Z_2$

$\pi_{26}^3=\{\bar{\alpha}\}\approx Z_4$

$\pi_{27}^4=\{E\bar{\alpha}\}\oplus\{\nu_4\circ\bar{\kappa}_7\}\approx Z_4\oplus Z_8$

$\pi_{28}^5=\{\nu_5\circ\bar{\kappa}_8\}\oplus\{\bar{\rho}'''\}\oplus\{\phi_5\}\approx Z_8\oplus Z_2\oplus Z_2$

$\pi_{29}^6=\{\nu_6\circ\bar{\kappa}_9\}\oplus\{\bar{\rho}''\}\oplus\{\phi_6\}\oplus\{\Delta(\lambda),\Delta(\xi)\}\approx Z_8\oplus Z_4\oplus Z_2\oplus(Z_8\oplus Z_4)$

$\pi_{30}^7=\{\nu_7\circ\bar{\kappa}_{10}\}\oplus\{\bar{\rho}'\}\oplus\{\phi_7\}\oplus\{\bar{\kappa}_7\circ\nu_{27}-\nu_7\circ\bar{\kappa}_{10}\}\oplus\{\sigma'\circ\sigma_{14}\circ\mu_{21}\}\oplus\{\sigma'\circ\omega_{14}\}$
$\approx Z_8\oplus Z_8\oplus Z_2\oplus Z_2\oplus Z_2\oplus Z_2$

$\pi_{31}^8=\{\nu_8\circ\bar{\kappa}_{11}\}\oplus\{E\bar{\rho}'\}\oplus\{\phi_8\}\oplus\{\bar{\kappa}_8\circ\nu_{28}-\nu_8\circ\bar{\kappa}_{11}\}\oplus\{E\sigma'\circ\sigma_{15}\circ\mu_{22}\}\oplus\{E\sigma'\circ\omega_{15}\}$
$\oplus\{\sigma_8^2\circ\mu_{22}\}\oplus\{\sigma_8\circ\omega_{15}\}\oplus\{\sigma_8\circ\eta^{*\prime}\}$
$\approx Z_8\oplus Z_8\oplus Z_2\oplus Z_2\oplus Z_2\oplus Z_2\oplus Z_2\oplus Z_2\oplus Z_2$

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***) This result was obtained independently by M. G. Barratt and M. Mahowald.