158. *Fundamental Solution of Partial Differential Operators of Schrödinger's Type. II*

The Space-Time Approach

By Daisuke FUJIWARA
Department of Mathematics, University of Tokyo
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§1. **Introduction.** In the previous note [2] we constructed the fundamental solution of $i\nu\dfrac{\partial}{\partial t}+1/2\Delta$, where Δ is the Laplace operator associated with a Riemannian metric $ds^2=\sum_{ij} g_{ij}(x)dx_i dx_j$ in R^n satisfying some conditions. There we made use of discussions of classical orbits in the phase space. In this note discussing in the spacetime, we shall construct the fundamental solution of $\nu i\dfrac{\partial}{\partial t}+\Delta$, $\nu>0$. This will be closer to the original Feynman's idea [1]. Assumptions will be found in §2 and results will be found in §4. In §3 we shall construct parametrix. The outline of proof will be given in §5. The main Lemma proof of which is too long to be presented in this short note will be proved in the subsequent paper [3].

§2. **Assumptions.** Let $|x-y|$ be the Euclidean distance from y to x and $r(x,y)$ be the geodesic distance from y to x. Our assumptions are the following ones:

(A–I) for any two points x, y in R^n, there exists unique geodesic joining x to y.

(A–II) the metric ds^2 coincides with the Euclidean metric outside compact set K.

(A–III) there exists a constant $C>0$ such that

$$|\operatorname{grad}_x (r^2(x,y)-r^2(x,z))| \geqq C\,|y-z|. \tag{1}$$

(A–IV) for any multi-indices α with $|\alpha|\geqq 2$, there exists a constant $C>0$ such that

$$\left|\left(\frac{\partial}{\partial x}\right)^{\alpha}(r^2(x,y)-r^2(x,z))\right| \leqq C\,|y-z|. \tag{2}$$

§3. **Parametrix.** We make use of the parametrix of the form

$$E_N(t,x,y)=(\nu/4\pi t i)^{1/2n}\exp(i\nu r^2(x,y)/4t)e(t,x,y), \tag{3}$$

$$e(t,x,y)=\sum_{j=0}^{N}(it/\nu)^j e_j(x,y). \tag{4}$$

If we use geodesic polar coordinates with center at y, the function