# 155. *Note on Pure Subsystems*

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**1.** By a *right S-system* $M_S$ over a semigroup $S$ we mean a set $M$ together with a mapping $(x, a) \rightarrow xa$ of $M \times S$ into $M$ satisfying

$$x(ab) = (xa)b$$

for all $x \in M$ and $a, b \in S$. A non-empty subset $N$ of a right $S$-system $M_S$ is called an *S-subsystem* of $M_S$ if $NS \subseteq N$. An $S$-subsystem $N$ of a right $S$-system $M_S$ is called *R-pure* in $S$ if

$$N \cap Ma = Na$$

for all $a \in S$. Since the inclusion $\supseteq$ is true for every $S$-subsystem $N$ of $M_S$, the essential requirement is

$$N \cap Ma \subseteq Na$$

for all $a \in S$. A right $S$-system $M_S$ is called *R*-pure* if every $S$-subsystem of $M_S$ is $R$-pure in $S$.

In [3] the author proved that for a semigroup $S$ with an identity the following conditions are equivalent:

(1) $S$ is regular.

(2) Every unital right $S$-system $M_S$ is $R^*$-pure.

(3) $S$ is $R^*$-pure.

In this note we shall give another properties of pure $S$-subsystems of a right $S$-system $M_S$ over a semigroup $S$. For the terminology not defined here we refer to the book by A. H. Clifford and G. B. Preston [1].

**2.** A subsemigroup $B$ of a semigroup $S$ is called a *bi-ideal* of $S$ if $BSB \subseteq B$. We denote by $[b]$ the principal bi-ideal of a semigroup $S$ generated by $b$ in $S$, that is,

$$[b] = b \cup b^2 \cup bSb.$$

First we give the following.

**Theorem 1.** *For an S-subsystem N of a right S-system $M_S$ over a semigroup S the following conditions are equivalent:*

(1) *$N$ is R-pure in $S$.*

(2) $N \cap MB = NB$ *for all bi-ideals $B$ of $S$.*

(3) $N \cap M[b] = N[b]$ *for all $b \in S$.*

**Proof.** First we assume that $N$ is $R$-pure in $S$. Let $B$ be any bi-ideal of $S$ and $p = qb$ $(p \in N, q \in M, b \in B)$ any element of $N \cap MB$. Then we have

$$p = qb \in N \cap Mb = Nb \subseteq NB$$