24. *The Local Maximum Modulus Principle for Function Spaces*

By Yukio HIRASHITA*) and Junzo WADA**)

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The local maximum modulus principle for function algebras due to H. Rossi [5] is well-known. The purpose of this paper is to consider the principle for function spaces, more correctly speaking, for function systems. In § 1, for any function system $\mathcal{F}$, we define the $LMM(\mathcal{F})$-boundary which plays the same rôle as the Shilov boundary in the Rossi's principle. In §§ 2 and 3, properties of the $LMM(\mathcal{F})$-boundary and relations between the Rossi's principle and ours are discussed.

§ 1. **The *LMM*-boundary.** Let X be a compact Hausdorff space. For any subset S in $X, \dot{S}$ denotes the topological boundary of S, i.e., $\dot{S}=\bar{S}\backslash S^i$, where $\bar{S}$ and S^i are the closure and the interior of S in X respectively.

Let $\mathcal{F}$ be a family of complex-valued bounded continuous functions defined on subsets of X. We denote the domain of f by $D(f)$ $(f \in \mathcal{F})$. $\mathcal{F}$ is said to be a *function system* on X if $\mathcal{F}$ has the following properties:

(1) If $f, g \in \mathcal{F}$ and α, β are complex numbers, then $\alpha f+\beta g$ (defined on $D(f) \cap D(g)$) belongs to $\mathcal{F}$.

(2) $\mathcal{F}_X=\{f \in \mathcal{F}: D(f)=X\}$ separates points of X and contains constant functions.

Let $\mathcal{F}$ be a function system on X. We will say that a subset E of X satisfies the *$LMM(\mathcal{F})$-principle* if $\|f\|_{\dot{U}}=\|f\|_U$ for any open subset U in X with $U \cap E=\phi$ and for any $f \in \mathcal{F}$ with $D(f)\supset \bar{U}$, where $\|f\|_P=\sup_{x\in P}|f(x)|$ for any P ($\|f\|_\phi=0$ for the empty set ϕ).

We shall first show that there exists the smallest one F_0 among non-void[1] closed subsets which satisfy the $LMM(\mathcal{F})$-principle. Such set F_0 is called the *$LMM(\mathcal{F})$-boundary* and we write $F_0=LMM(\mathcal{F})$.

Theorem 1.1. *For any function system $\mathcal{F}$, there exists the $LMM(\mathcal{F})$-boundary.*

Proof. Let $\mathcal{P}=\{F_\lambda\}_{\lambda\in\Lambda}$[2] be the family of all (non-void) closed subsets in X which satisfy the $LMM(\mathcal{P})$-principle. We define a partial order $\succ$ in Λ as follows: $\lambda \succ \mu$ if and only if $F_\lambda \supset F_\mu$. It is not hard to

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*) Kyushu University, Fukuoka.
**) Waseda University, Tokyo.

1) The empty set ϕ does not satisfy the $LMM(\mathcal{F})$-principle.
2) $\mathcal{P}$ is non-void, because $\mathcal{P} \ni X$.