66. *A Remark on Picard Principle. II*

By Mitsuru NAKAI
Department of Mathematics, Nagoya Institute of Technology

(Comm. by Kôsaku YOSIDA, M. J. A., May 9, 1975)

The purpose of this note is to announce two results on the Picard principle in the unpublished papers [10] and [11] which will be published later elsewhere.

1. A nonnegative locally Hölder continuous function $P(z)$ on $0<|z|\leq 1$ will be referred to as a *density* on $\Omega: 0<|z|<1$. The *elliptic dimension* of a density P on Ω at $\delta: z=0$, $\dim P$ in notation, is the dimension of the half module $\mathscr{P}$ of nonnegative solutions of $\Delta u=Pu$ on Ω with vanishing boundary values on $\partial\Omega: |z|=1$. More precisely, let $\mathscr{P}_1$ be the convex set of $u\in\mathscr{P}$ with the normalization $\int_0^{2\pi}[u_r(re^{i\theta})]_{r=1}d\theta=-1$. Then we define

$$\dim P=\sharp(ex[\mathscr{P}_1]) \tag{1}$$

where $ex[\mathscr{P}_1]$ is the set of extreme points of $\mathscr{P}_1$ and $\sharp$ denotes the cardinal number. We say that the *Picard principle* is valid for P at δ if $\dim P=1$. The study of Picard principle is initiated by Picard, Stozek, and Bouligand. The present formulation as well as the first step to a systematic study is taken by Brelot [1]. For further developments and related works we refer to Heins [3], Ozawa [12], [13], Hayashi [2], Nakai [6]-[9], Kawamura-Nakai [5], among others. The first of our announcements is the following practical test of the Picard principle [10]:

Theorem. *The Picard principle is valid at δ for any finite density P on Ω, i.e. for any density P with the following property*

$$\int_\Omega P(z)dxdy<\infty \qquad (z=x+iy). \tag{2}$$

We shall give an outline of the proof of the above in no. 4. The proof is based on a general theory on the Picard principle originally obtained by Heins [3] and Hayashi [2]. We state this in the next no.

2. Let Ω be an *end* of an m dimensional $(m\geq 2)$ C^∞ Riemannian manifold, i.e. Ω is a manifold with a compact smooth relative boundary $\partial\Omega$ and a single ideal boundary compact δ. A typical example is the one in no. 1: $\Omega: 0<|z|<1$, $\partial\Omega: |z|=1$, $\delta: z=0$. Consider an elliptic differential operator L on $\bar{\Omega}$ given by

$$Lu(x)=\Delta u(x)+b(x)\cdot\nabla u(x)+c(x)u(x) \tag{3}$$

for $u\in C^2(\Omega)$, where Δ is the Laplace-Beltrami operator on the