113. Normalized Series of Prestratified Spaces

Complex Analytic De Rham Cohomology. IV

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In this note we introduce,¹⁾ for analytic varieties, a type of series of prestratified spaces, which we call a *normalized series of prestratified spaces* (or simply a *normalized series*, when there is no fear of confusions). We also state an existence theorem on such a series. We stated two basic quantitative properties of analytic varieties in $[4]_2$. It is this notion of normalized series that constitutes basis of the discussions for the results in $[4]_2$.

Basic ideas. Let V be an algebraic or analytic variety.²⁾ The basic theorems: Weierstrass's preparation theorem and Noether's normalization theorem represent the variety V as a (finite) ramified covering of an another variety V', which has simpler properties than V. In both theorems the study of the ramification locus W of the covering map $\pi: V \rightarrow V'$ has important meanings for the study of the variety V. Of course, dim $W < \dim V$, and we may say that the above theorems enable us *inductive discussions* of varieties on the dimension of varieties in question. We note, moreover, that the above theorems attach to the given variety V a set of functions, which is basic in the study of the variety V.

Now our hope in introducing the notion of normalized series is to systematize ideas³⁾ in the above theorems (and methods of ramified maps in general): Let V be an analytic variety. Then a *normalized series attached to* V consists of series \Re of varieties, prestratified spaces, \cdots and \Im of collections of analytic functions (cf. n. 2). Varieties and strata appearing in the series \Re are basically related to each other by ramified maps (arising naturally from the series \Re).

By attaching to the given variety V a *series* of varieties, prestratifications, \cdots instead of a single variety (as in standard treatments of basic theorems mentioned above), we can discuss, systematically, the variety V inductively on the dimension of varieties, \cdots (appearing

¹⁾ We use the same notions and notations as in $[4]_1$, $[4]_2$ and $[4]_8$. In particular we use the notion of prestratified spaces in the sense in $[4]_8$.

²⁾ Except the part explaining basic ideas in the introduction, analytic varieties and analytic functions are always *real* analytic ones.

³⁾ Ideas understood as explained just before.