146. A Vietoris Theorem in Shape Theory

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1. Introduction. In this paper the notion of shape is understood in the sense of Mardešić [2] and our approach to shape theory (cf. [5], [6]) will be used.

Our approach enables us to define the k-th homotopy pro-group $\pi_k\{(X, x_0)\}$ of a pointed topological space (X, x_0). The homotopy pro-groups play the central role in the Whitehead theorem in shape theory.

Theorem 1.0 (Morita [6]). *Let $f:(X, x_0)\to(Y, y_0)$ be a shape morphism of pointed connected topological spaces. If the induced morphism $\pi_k(f):\pi_k\{(X, x_0)\}\to\pi_k\{(Y, y_0)\}$ of homotopy pro-groups is an isomorphism for $1\leqq k\leqq n$ and an epimorphism for $k=n+1$ where $n+1=\max(1+\dim X, \dim Y)<\infty$, then f is a shape equivalence.*

In this paper, by using homotopy pro-groups we shall formulate a Vietoris theorem in shape theory as follows.

Theorem 1.1. *Let $f:(X, x_0)\to(Y, y_0)$ be a closed continuous map from a pointed metrizable space (X, x_0) onto a pointed topological space (Y, y_0) such that $f^{-1}(y)$ is approximatively k-connected for every point y of Y and for $0\leqq k\leqq n$. Then the induced morphism $\pi_k(f):\pi_k\{(X, x_0)\}\to\pi_k\{(Y, y_0)\}$ of homotopy pro-groups is an isomorphism for $1\leqq k\leqq n$ and an epimorphism for $k=n+1$.*

The following is a direct consequene of Theorems 1.0 and 1.1 as far as X is connected or locally connected.

Theorem 1.2. *Let f be the same as in Theorem* 1.1. *If, in addition,* $\dim X\leqq n$ *and* $\dim Y\leqq n+1$, *then f is a shape equivalence.*

As is quoted in [3, p. 319], in the first version of [5] we defined the k-th shape group $\underline{\pi}_k(X, x_0)$ of a pointed topological space (X, x_0) to be the inverse limit of $\pi_k\{(X, x_0)\}$. For metric compacta M. Moszyňska [8] proved that the shape groups are naturally isomorphic to the fundamental groups in the sense of K. Borsuk. Thus, our Theorem 1.1 extends a result for metric compacta which was announced by S. Bogaty [1] and proved by K. Kuperberg [9].

2. Preliminaries. Let X be a metrizable space. Then there is a metric space X_0 which is an ANR for metric spaces and contains X as its closed subset. Let $f:(X, x_0)\to(Y, y_0)$ be a closed continuous map from (X, x_0) onto a pointed topological space (Y, y_0). Then the collection $\{f^{-1}(y)\mid y\in Y\}\cup\{\{x\}\mid x\in X_0-X\}$ of subsets of X_0 defines an upper