172. *On Holomorphically induced Representations of Split Solvable Lie Groups*

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We shall give an answer to three open problems for holomorphically induced representations of split solvable Lie groups.

1. Let G be a simply connected split solvable Lie group with Lie algebra $\mathfrak{g}$, f a linear form on $\mathfrak{g}$, $\mathfrak{h}$ a positive polarization of $\mathfrak{g}$ at f, $\rho(f, \mathfrak{h})$ the holomorphically induced representation of G constructed from $\mathfrak{h}$ and let $\mathcal{H}(f, \mathfrak{h})$ be the space of $\rho(f, \mathfrak{h})$ [1]. In this note, we find a necessary and sufficient condition on $(f, \mathfrak{h})$ for the non-vanishing of $\mathcal{H}(f, \mathfrak{h})$. We then show that $\rho(f, \mathfrak{h})(\neq 0)$ is irreducible if and only if the Pukanszky condition is satisfied, and in this case $\rho(f, \mathfrak{h})$ is independent of $\mathfrak{h}$. For reducible $\rho(f, \mathfrak{h})$, we describe its decomposition into irreducible components.

The details will appear elsewhere.

2. For a real vector space V, we denote its dual by V^*. Let $\mathfrak{d}=\mathfrak{h}\cap\mathfrak{g}$, $\mathfrak{e}=(\mathfrak{h}+\bar{\mathfrak{h}})\cap\mathfrak{g}$ and let $\mathfrak{b}=\mathfrak{d}\cap\ker f$. $\mathfrak{d}$ and $\mathfrak{b}$ are ideals of $\mathfrak{e}$. Let $\tilde{\mathfrak{e}}=\mathfrak{e}/\mathfrak{b}$, $\mathfrak{z}=\mathfrak{d}/\mathfrak{b}$, $\pi: \mathfrak{e}\to\tilde{\mathfrak{e}}$ the natural projection, $f_0=f|\mathfrak{e}\in\mathfrak{e}^*$, $\tilde{\mathfrak{h}}=\pi(\mathfrak{h})$ and let $\tilde{f}\in(\tilde{\mathfrak{e}})^*$ such that $\tilde{f}\circ\pi=f_0$. We denote by $P^+(f, \mathfrak{g})$ the set of positive polarizations of $\mathfrak{g}$ at f. Then, as a corollary of the fundamental theorem for normal Kähler algebras [3], we have the following theorem.

Theorem 1. *$\tilde{\mathfrak{e}}$ can be decomposed into a semi-direct sum*

$$\tilde{\mathfrak{e}}=\mathfrak{n}+\mathfrak{m},\ \mathfrak{m}: \textit{subalgebra},\ \mathfrak{n}: \textit{ideal},$$

and this decomposition satisfies the following conditions:

Let $\mathfrak{h}_1=\tilde{\mathfrak{h}}\cap\mathfrak{n}^C$, $\mathfrak{h}_2=\tilde{\mathfrak{h}}\cap\mathfrak{m}^C$, $\tilde{f}_1=\tilde{f}|\mathfrak{n}\in\mathfrak{n}^*$ and let $\tilde{f}_2=\tilde{f}|\mathfrak{m}\in\mathfrak{m}^*$.

a) *$\mathfrak{n}$ is a Heisenberg algebra with center $\mathfrak{z}$ and $\mathfrak{h}_1\in P^+(\tilde{f}_1, \mathfrak{n})$.*

b) *$\mathfrak{h}_2\in P^+(\tilde{f}_2, \mathfrak{m})$ and $\mathfrak{h}_2+\bar{\mathfrak{h}}_2=\mathfrak{m}^C$, $\mathfrak{h}_2\cap\mathfrak{m}=\{0\}$. We define the linear operator j on $\mathfrak{m}$ by $j(X)=-iX$ if $X\in\mathfrak{h}_2$, $j(X)=iX$ if $X\in\bar{\mathfrak{h}}_2$. Then $(\mathfrak{m}, j)$ is a narmal j-algebra.*

Note that $\mathfrak{n}$ or $\mathfrak{m}$ may be $\{0\}$.

3. We put $S(X, Y)=\tilde{f}_2([X, jY])$ for $X, Y\in\mathfrak{m}$.

Theorem 2 (Pjateckiĭ-Šapiro [4]). *Let $\mathfrak{a}$ be the orthogonal complement of $\eta=[\mathfrak{m}, \mathfrak{m}]$ with respect to the form S. $\mathfrak{a}$ is a commutative subalgebra of $\mathfrak{m}$, $\mathfrak{m}=\mathfrak{a}+\eta$, and the adjoint representation of $\mathfrak{a}$ on η is real diagonalizable. Thus, we have a decomposition of η into root spaces: $\eta=\sum\eta^\alpha$, where $\alpha\in\mathfrak{a}^*$ and $\eta^\alpha=\{X\in\eta;\ [A, X]=\alpha(A)X$ for all $A\in\mathfrak{a}\}$. Let $\{\eta^{\alpha_i}\}$, $1\leq i\leq r$ be those root spaces η^α for which $j(\eta^\alpha)\subset\mathfrak{a}$.*