168. *The Isometry Groups of Compact Manifolds with Non-positive Curvature*[*)]

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Let M be an n-dimensional compact connected Riemannian manifold with negative Ricci curvature. Then a classical theorem of Bochner says that there exist no non-trivial Killing vector fields on M. And hence the order of the isometry group $I(M)$ of M is finite. Relating to this theorem, T. Frankel obtained the following:

Let M be a compact Riemannian manifold with non-positive sectional curvature $K_\sigma \leqq 0$ and with negative Ricci curvature. If $f: M \to M$ is an isometry which is continuously homotopic to the identity map, then f is the identity, see [1]. This result was extended by H.B. Lawson and S. T. Yau in a more general situation. That is

Theorem ([5; Theorem 4, p. 225]). *Let M be a compact Riemannian manifold with non-positive sectional curvature and Ricci curvature negative at some point of M. If $f: M \to M$ is an isometry continuously homotopic to the identity, then f is the identity.*

As a corollary of this theorem, we easily have

Lemma 1. *Let M be a manifold as in the theorem of Lawson and Yau. If $f: M \to M$ is an isometry such that $d(p, f(p)) < d(p, C(p))$ for all point $p \in M$, then f is the identity.*

Here d is the distance function of M induced from the Riemannian metric and $C(p)$ the cut locus of p in M.

Now, for such manifolds as in the theorem of Bochner or Lawson and Yau, it is natural to ask whether we can estimate the order of the isometry group $I(M)$ by using the geometrical terms of M, for example, the diameter, the injectivity radius, the sectional curvature and so on. To this problem, H. C. Im Hof gave an estimation of order of $I(M)$ for a manifold with the sectional curvature K_σ satisfying $-b^2 \leqq K_\sigma \leqq -a^2 < 0$, $0 < a \leqq b$. In his argument, the assumption that M is of negative curvature is essential.

In this note, we will give an estimation of the order of $I(M)$ for manifolds as in the theorem of Lawson and Yau in a different way from the one in H. C. Im Hof's theorem. The author thanks Prof. T. Otsuki for his kind advices.

Let M be a compact Riemannian manifold. For a point $p \in M$,

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