44. *On a Conjecture of Regge and Sato on Feynman Integrals*

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The purpose of this note is to show that any (unrenormalized) Feynman integral should satisfy a holonomic system of linear differential equations. This result gives an affirmative answer to the conjecture given in an illuminating report by Regge [7], who first understood and emphasized the importance of the role of the differential equations in the investigation of Feynman integrals. This important property of the Feynman integral has also been conjectured and proved in simple cases by Sato [8] independently and in a little different context. See also Barucchi-Ponzano [1], Kawai-Stapp [5] and references cited there. Note that Kawai-Stapp [5] discusses the S-matrix itself, not the individual Feynman integrals, as Sato [8] proposes. The characteristic variety of the holonomic system discussed here enjoys a nice physical interpretation, as is shown by Kashiwara-Kawai-Stapp [4]. In this note we discuss the generalized Feynman integral after Speer [10]. For simplicity we assume that all relevalent particles are spinless. However, we do not necessarily assume that their masses are different from zero.

Renormalized integrals will be discussed in our subsequent papers.

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Our arguments rely on following lemmas. (Cf. Kashiwara-Kawai [3], Kashiwara [2] and Kashiwara-Kawai-Stapp [4].)

Lemma 1. *Let $\varphi_j(x)$ $(j=1, \cdots, d)$ and $f_l(x)$ $(l=1, \cdots, N)$ be a real valued real analytic functions defined on a real analytic manifold M. Denote by X a complexification of M. Denote by Y the variety defined by $\{x \in X; \varphi_1(x)=\cdots=\varphi_d(x)=0\}$. Assume that Y has codimension d in X and that Y is irreducible and non-singular except for proper analytic subset Y_{sing} of Y. Assume that $f_l|_Y \not\equiv 0$ $(l=1, \cdots, N)$ and that*

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