75. *A Counterexample for the Local Analogy of a Theorem by Iwasawa and Uchida*

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Let $\boldsymbol{Q}$ be the rational number field, $\bar{\boldsymbol{Q}}$ the algebraic closure of $\boldsymbol{Q}$ and k_1, k_2 two finite extensions of $\boldsymbol{Q}$ contained in $\bar{\boldsymbol{Q}}$ such that $\mathrm{Gal}(\bar{\boldsymbol{Q}}/k_1) \cong \mathrm{Gal}(\bar{\boldsymbol{Q}}/k_2)$ as topological groups. As K. Iwasawa and K. Uchida have independently proved (cf. [2], [6]), k_1 and k_2 are conjugate over $\boldsymbol{Q}$. In this paper we prove that the analogy for local number fields is not valid: Let p be a prime number, $\boldsymbol{Q}_p$ the field of p-adic numbers, $\bar{\boldsymbol{Q}}_p$ the algebraic closure of $\boldsymbol{Q}_p$. Then there are finite extensions K_1, K_2 of $\boldsymbol{Q}_p$ contained in $\bar{\boldsymbol{Q}}_p$ such that $\mathrm{Gal}(\bar{\boldsymbol{Q}}_p/K_1) \cong \mathrm{Gal}(\bar{\boldsymbol{Q}}_p/K_2)$ as topological groups, and that K_1 and K_2 are not conjugate over $\boldsymbol{Q}_p$.

§1. **Preliminaries.** Let K be a local field, i.e. a commutative field which is complete with respect to a discrete valuation, and L/K a finite Galois extension with $G = \mathrm{Gal}(L/K)$ such that the extension of their residue class fields is separable. Let v_L be the normalized discrete valuation of L, and put $A_L = \{a \in L \mid v_L(a) \geqq 0\}$ and $G_x = \{s \in G \mid v_L(s(a) - a) \geqq x+1$ for all $a \in A_L\}$ for $x \geqq -1$. The function $\varphi_{L/K}(t)$ for $t \geqq -1$ is given by

$$\varphi_{L/K}(t) = \int_0^t \frac{dx}{(G_0 : G_x)}.$$

Let $\psi_{L/K}$ be the inverse function of $\varphi_{L/K}$ and put $G^x = G_{\psi_{L/K}(x)}$. A real number $x \geqq -1$ is called a ramification number of L/K (an upper ramification number of L/K, respectively) if $(G_x : \bigcup_{\varepsilon>0} G_{x+\varepsilon}) > 1$ (if $(G^x : \bigcup_{\varepsilon>0} G^{x+\varepsilon}) > 1$, respectively). When L/K has only one ramification number x, x is also the only one upper ramification number of L/K and vice versa. L/K is totally ramified if and only if $G = G_0$.

Lemma (cf. [4], p. 197 and p. 198). *Let K_i/K be a cyclic extension of degree p with only one upper ramification number t_i for $i=1,2$, where p is the characteristic of the residue class field of K. Assume that the residue class field extension of K_i/K is separable. Put $M = K_1K_2$. If $t_1 \neq t_2$, M/K_2 is a cyclic extension of degree p with only one upper ramification number $\psi_{K_2/K}(t_1)$.*

In the above situation, we remark that M/K_2 is totally ramified if K_1/K is totally ramified.

Let ζ_n be a primitive n-th root of 1 in $\bar{\boldsymbol{Q}}_p$.